THE RELEVANCE OF THE TRIAXIAL TEST TO THE SOLUTION OF STABILITY PROBLEMS

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1. INTRODUCTION

The purpose of the present paper is to show how the actual properties of cohesive soils measured in the standard undrained, consolidated-undrained and drained triaxial test are applied to the solution of the more important classes of stability problem encountered by the practicing engineer. The failure criteria chosen and the shear parameters by which they are expressed are those found most convenient and most appropriate to the methods of stability analysis used. The relation of the practical shear parameters to the more basic shear parameters proposed, for example, by Hvorslev (1937)[3] is outside the scope of the present paper and is discussed elsewhere (Skempton and Bishop, 1954; Bjerrum, 1954 b).

The practical shear parameters serve to take full account of the principal differences between cohesive soils and other structural materials, such as the dependence of strength on the state of stress and on the conditions of drainage.

Although the purpose of the paper is to present the logical relationship of the various standard tests to the different classes of stability problem, attention must also be drawn to the various limitations of the apparatus in which the triaxial test is usually performed. These include non-uniformity of stress and strain particularly at large deformations, and the inability of the apparatus to simulate the changes in direction of the principal stresses which occur in many practical problems. To enable the quantitative importance of these limitations to be seen in perspective, emphasis is laid on the direct correlation between laboratory tests and field observations of stability (or instability) wherever case records are available.

It may seem to the practicing engineer that many of his problems are too small in scale or are in soils too lacking in homogeneity to apply detailed quantitative methods of stability analysis. However, even for the application of semi-empirical rules it is important to determine into which class the stability problem falls.

2. THE PRINCIPLE OF EFFECTIVE STRESS

One of the main reasons for the late development of Soil Mechanics as a systematic branch of Civil Engineering has been the difficulty in recognising

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3. Items indicated thus, Hvorslev (1937), refer to corresponding entries listed alphabetically in the Appendix Bibliography.

that the difference between the shear characteristics of sand and clay lies not so much in the difference between the frictional properties of the component particles as in the very wide difference—about one million times—in permeability. The all-round component of a stress change applied to a saturated clay is thus not effective in producing any change in the frictional component of strength until a sufficient time has elapsed for water to leave (or enter), so that the appropriate volume change can take place.

The clarification of this situation did not begin until the discovery of the principle of effective stress by Terzaghi (1923 and 1932) and its experimental investigation by Rendulic (1937). An examination of current design methods might suggest that the impact of Terzaghi's discovery had yet to be fully felt.

For soil having a single fluid, either water or air, in the pore space, the principle of effective stress may be expressed in relation both to volume change and to shear strength:

(a) The change in volume of an element of soil depends, not on the change in total normal stress applied, but on the difference between the change in total normal stress and the change in pore pressure. For an equal all-round change in stress this is expressed quantitatively by the expression:

$$\Delta V / V = - C_c(\Delta\sigma - \Delta u) \tag{1}$$

where $\Delta V/V$ denotes the change in volume per unit volume of soil,
$\Delta\sigma$ denotes the change in total normal stress,
Δu denotes the change in pore pressure,
and C_c denotes the compressibility of the soil skeleton for the particular stress range considered.

It may be noted in passing that this equation shows that a decrease in pore pressure at constant total stress is as effective in producing a volume change as an increase in total stress at constant pore pressure, a fact which is confirmed by field experience.

(b) The maximum resistance to shear on any plane in the soil is a function, not of the total normal stress acting on the plane, but of the difference between the total normal stress and the pore pressure. This may be expressed quantitatively by the expression:

$$\tau_f = c' + (\sigma - u) \tan \Phi' \tag{2}$$

where τ_f denotes the shear stress on the plane at failure,
c' denotes the apparent cohesion, } in terms of effective
Φ' denotes the angle of shearing resistance, } stress.
σ denotes the total stress on the plane considered,[a]
and u denotes the pore pressure.

In both cases the effective normal stress is thus the stress difference σ - u, usually denoted by the symbol σ'.

The validity of the principle of effective stress has been amply confirmed, for saturated soils, by the experimental work of Rendulic (1937), Taylor (1944), Bishop and Eldin (1950),[b] and Laughton (1955)[b]; and indirectly by the field records referred to in later sections of this paper. For partly saturated soils, however, a more general form of expression must be used, since the pore space contains both air and water which may be in equilibrium at widely different pressures, due to surface tension. A tentative expression has been suggested for the effective stress under these conditions (Bishop, 1959[b]; 1960), of the form:

$$\sigma' = \sigma - u_1 + x\,(u_1 - u_2) \tag{3}$$

where u_1 denotes the pressure in the air in the pore space,
u_2 denotes the pressure in the water in the pore space,
and x is a parameter closely related to the degree of saturation S and varying from unity in saturated soils to zero in dry soils.

The parameter x and its values under various soil conditions are discussed in more detail elsewhere (Bishop, 1960; Bishop, Alpan, Blight, and Donald, 1960). It may be noted in passing that for a given soil condition, the value of x measured in relation to shear strength may differ from its value measured in relation to volume change. However, the large positive pore pressures likely to lead to instability in rolled fills will in general only occur if the degree of saturation is high, where x may be equated to unity with little error. The additional complication of observing or predicting pore air pressure may therefore hardly be justified in such cases.

In most stability problems the magnitude of the body forces and of the applied loads is known quite accurately. It is in the magnitude of the shear strength that the main uncertainty lies and it is therefore useful to examine the variables controlling the value of τ_f in equation (2).

The magnitude of the total normal stress σ on a potential slip surface may be estimated with reasonable accuracy from considerations of statics. The shear parameters c' and Φ' are properties which depend primarily on the soil type and to a limited extent on stress history (see Table I in section 6). Provided representative samples are taken and tested in the appropriate stress range, little error need arise in evaluating c' and Φ'. This aspect of any investigation does, however, call for sound judgment and a knowledge of geology.

It is in the prediction of the value of the pore pressure u that in many problems the greatest uncertainty lies. The development of cheap and reliable field devices for measuring pore pressure in soils of low permeability[c] has, however, transformed the situation as far as the practicing engineer is concerned by enabling predictions to be checked and a control to be kept on stability during construction work.

Much of the uncertainty about the pore pressure prediction has arisen from a failure to distinguish between the two main classes of problem[d]:

(a) Problems where pore pressure is an independent variable and is controlled either by ground water level or by the flow pattern of impounded or underground water, for example, and

(b) Problems in which the magnitude of the pore pressure depends on the magnitude of the stresses tending to lead to instability, as in rapid construction or excavation in soils of low permeability.

a. Stresses and pressures are here considered as measured with respect to atmospheric pressure as zero (i.e. gauge pressures). The actual datum does not of course affect the value of the effective stress.

b. A treatment of the influence of contact area is given in these papers.

c. See, for example, Casagrande, 1949; U.S.B.R., 1951; Penman, 1956; Sevaldson, 1956; Kallstenius and Wallgren, 1956; Bishop, Kennard, and Penman, 1960.

d. This distinction is discussed in detail by Bishop (1952).

In problems which initially fall into class (b) the pore pressure distribution will change with time and at any point the pore pressure will either decrease or increase to adjust itself to the ultimate condition of equilibrium with the prevailing conditions of ground water level or seepage. The rate at which this adjustment occurs depends on the permeability of the soil (as reflected in its coefficient of consolidation) and on the excess pore pressure gradients, which depend both on the stress gradients and on the distance to drainage surfaces.

The least favourable distribution of pore pressure may occur either in the initial stage or at the ultimate condition or, in special cases, at an intermediate time depending, for example, on whether load is applied or removed, and on other specific details of the problem, as discussed in section 6.

3. PORE PRESSURE PARAMETERS

In slope stability problems the influence of pore pressure on the factor of safety is most conveniently expressed in terms of the ratio of the pore pressure to the weight of material overlying the potential slip surface. This ratio was used by Daehn and Hilf (1951) in the form of an overall ratio of the sum of resolved components of the pore pressure and of the weight of soil, to express the results of the stability analysis of four earth dams, based on the field measurement of pore pressure.

Bishop (1952 and 1954 b) showed that, for a slope in which the ratio of the pore pressure u to the vertical head[e] of soil γh above the element considered was a constant, the value of the factor of safety F decreased almost linearly with increase in pore pressure ratio $u/\gamma h$. Subsequent work by Bishop and Morgenstern (in course of preparation for publication) has shown that, both for pore pressures obtained from flow patterns (class (a) problems) and for those obtained as a function of stress (class (b) problems), the average value of the pore pressure ratio $u/\gamma h$ is the most convenient dimensionless parameter by which to express the influence of pore pressure stability (Fig. 1). The ratio is denoted r_u.

Where the pore pressure is independent of stress, its value is obtained directly from the ground water conditions or flow net and expressed as the average[f] value of r_u. In all other cases (class (b) problems) the ratio must either be obtained from field measurements or predicted from the observed relationships between pore pressure and stress change under undrained conditions and from the theory of consolidation. This in turn necessitates an estimate of the stress distribution within the soil mass.

For the inclusion of the laboratory results in the stability calculation it is convenient to express them in terms of pore pressure parameters. The development of these parameters[g] and their application to practical problems is described in detail elsewhere (Skempton, 1948 b; Bishop, 1952; Skempton, 1954; Bishop, 1954 a; Bishop and Henkel, 1957; Bishop and Morgenstern, 1960).

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e. γ is the average bulk density of the soil and h the vertical distance of the surface above the element.

f. Details of the averaging method are given by Bishop and Morgenstern, 1960.

g. Attention was drawn to the possibility of pore pressure changes in clays under the action of a deviator stress before it proved practicable to measure them (Terzaghi, 1925: Casagrande, 1934).

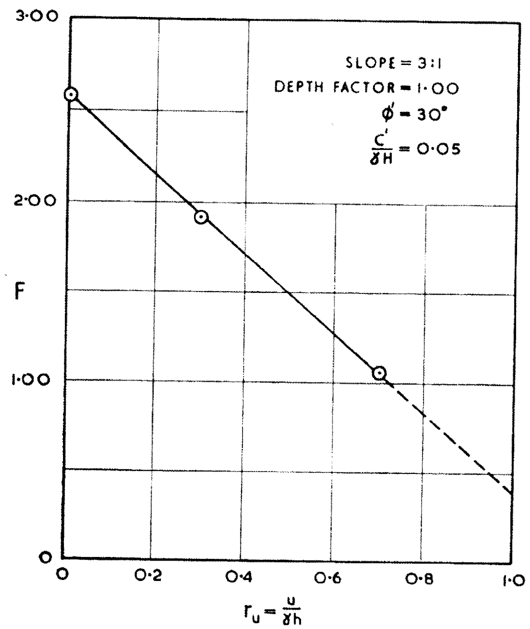

Fig. 1.—The linear relationship between factor of safety and pore pressure ratio for a slope or cut in cohesive soil.

For a change in stress under undrained conditions the change in pore pressure may be expressed as Δu, where

$$\Delta u = B\,[\Delta\sigma_3 + A\,(\Delta\sigma_1 - \Delta\sigma_3)] \tag{4}$$

where $\Delta\sigma_1$ denotes the change in major principal stress,
$\Delta\sigma_3$ denotes the change in minor principal stress,
(in both cases total stresses are considered)
and A and B denote the pore pressure parameters (Skempton, 1954).

Triaxial compression tests show that for fully saturated soils B = 1 to within practical limits of accuracy, and that the value of A depends on stress history and on the proportion of the failure stress applied. This is illustrated in Fig. 2. where the A values for normally and overconsolidated clay are given. The values of A at failure are seen to be very dependent on the over-consolidation ratio (defined as the ratio of the maximum consolidation pressure to which the soil has been subjected to the consolidation pressure immediately before the undrained test is performed).

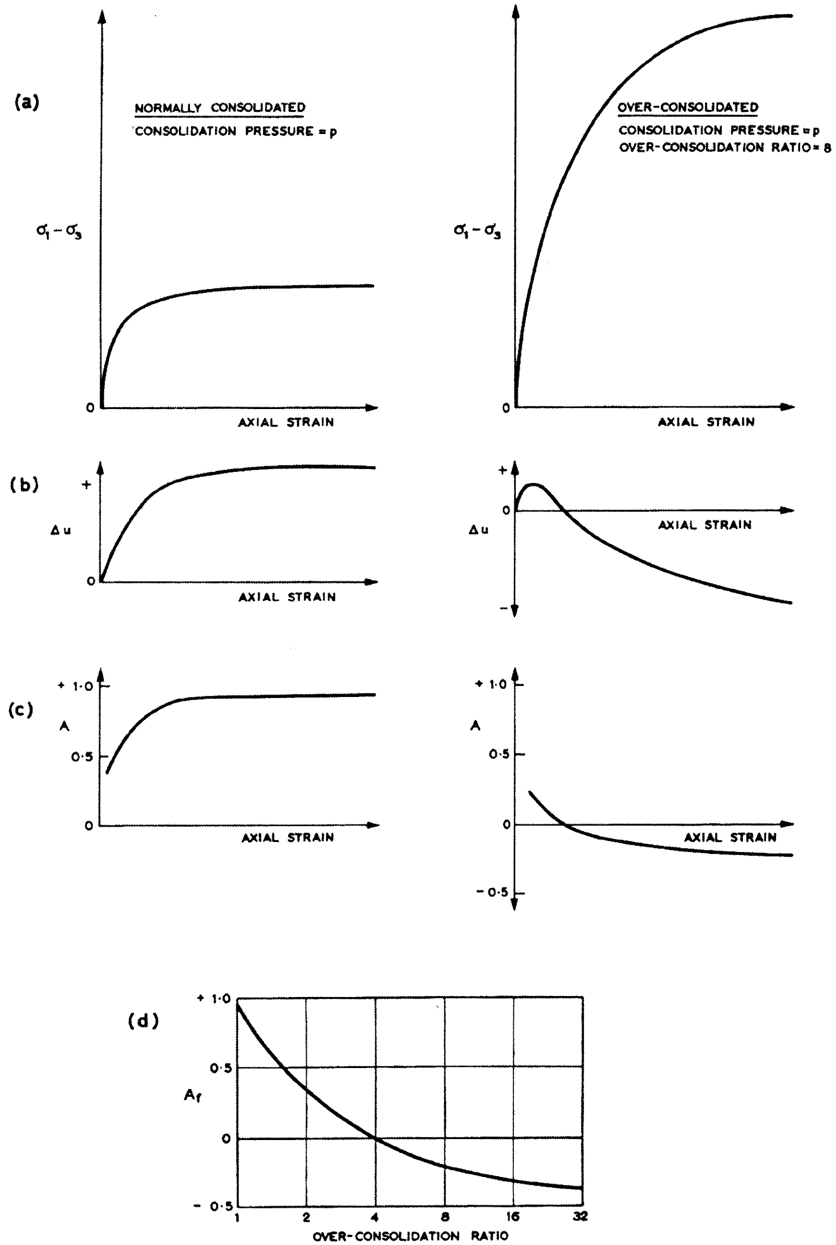

Fig. 2.--The dependence of the pore pressure parameter A on stress history.

For partly saturated soils the value of B lies between 0 and 1 depending on the degree of saturation and the compressibility of the soil skeleton. Typical values of A and B are used in section 6[h].

It should be noted that equation 4 takes no account of the change in the intermediate principal stress $\Delta\sigma_2$, or of possible changes in the directions of the principal stresses. In the majority of stability problems the conditions approximate to plane strain, in which the intermediate principal stress does not equal the minor principal stress as in the standard cylindrical compression test. Theoretical studies (Skempton, 1948 a and b; Hansen and Gibson, 1949; Bishop and Henkel, 1957) indicate that the form of the equation remains the same, but that the triaxial test underestimates the value of A, and the limited amount of test data so far available (Wood, 1958; Cornforth, 1960; Henkel, 1960) supports this view. Little is yet known about the influence of the rotation of the principal stresses on the value of A. The importance of these limitations in practice can at present only be assessed from the overall check with observed pore pressures in the field.

It should also be noted that the principle of superposition can be applied to pore pressure changes in soil only in a very restricted sense. Where the purpose of the test is the accurate prediction of pore pressure at states of stress other than failure, a more accurate result is obtained if the stress increments occurring in practice are closely followed in the test by making simultaneous changes in the values of both σ_1 and σ_3. The test result is then conveniently expressed in terms of the relationship between pore pressure and major principal stress, for the specified stress ratio, using the expression:

$$\Delta u = \bar{B} \, . \, \Delta\sigma_1 \tag{5}$$

The influence of stress ratio on this parameter is illustrated in Fig. 3 for a compacted earth fill.

It should be noted that the value of the parameter $\bar{B}$ only gives the change in pore pressure due to stress change under undrained conditions. The actual pore pressure depends also on the initial value u_0 before the stress change is made (Fig. 4), and is given by the expression:

$$u = u_0 + \Delta u \tag{6}$$

$$\text{i.e.} \quad u = u_0 + \bar{B} \, . \, \Delta\sigma_1 \tag{7}$$

In natural strata u_0 is determined from the initial ground water conditions, being positive below ground water level and negative above. In rolled fill the initial value is usually negative, reaching quite high values in cohesive soils placed at or below the optimum water content (Hilf, 1956; Bishop, 1960; Bishop, Alpan, Blight, and Donald, 1960).

In cases where no dissipation of pore pressure is assumed to occur, the pore pressure ratio r_u used in the stability analysis is directly related to $\bar{B}$:

$$r_u = u/\gamma h = 1/\gamma h \, . \, (u_0 + \bar{B}\Delta\sigma_1) \tag{8}$$

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h. The value of B is generally given with respect to changes in pore water pressure. A slightly different value relates the change in air pressure to the change in total stress.

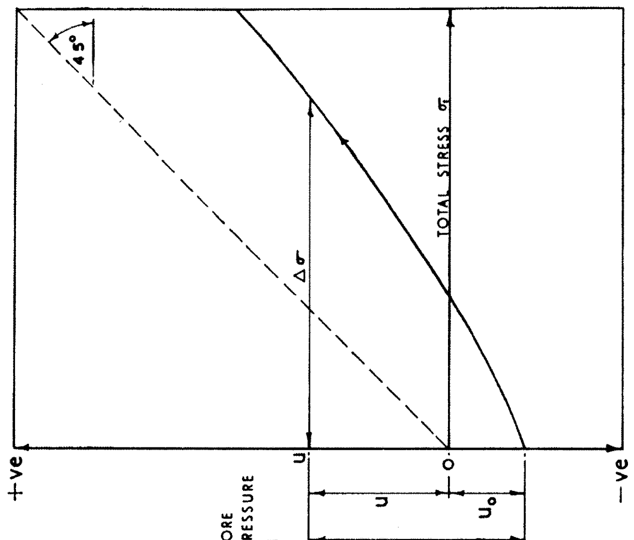

Fig. 4.—Pore pressure change under undrained test conditions.

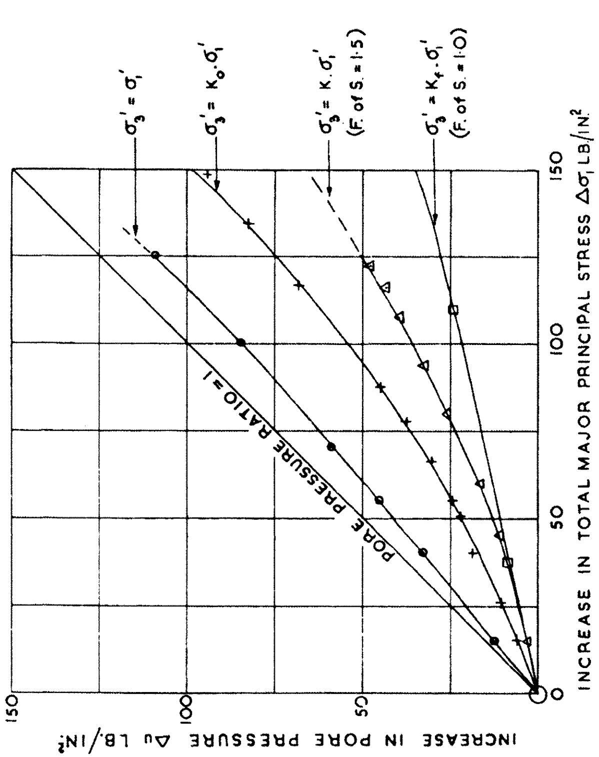

Fig. 3.—The influence of principal stress ratio on the pore pressure parameter $\bar{B}$: boulder clay compacted at optimum + 1%.

In the special case of the construction of earth fill embankments the average value of $\Delta\sigma_1$ along a potential slip surface approximates to γh (Bishop, 1952), and equation (8) becomes:

$$r_u = \bar{B} + u_0/\gamma h \tag{9}$$

For earth fills of low plasticity placed wet of the optimum the term $u_0/\gamma h$ is small, and a further approximation is sometimes used in preliminary design:

$$r_u = \bar{B} \tag{10}$$

Some typical examples of the use of pore pressure parameters are given in section 6.

4. STANDARD TYPES OF TRIAXIAL TEST

The type of triaxial test most commonly used in research work and in routine testing is the cylindrical compression test. A diagrammatic layout of the apparatus is given in Fig. 5.

The cylindrical specimen is sealed in a thin rubber membrane and subjected to fluid pressure. A load applied axially, through a ram acting on the top cap, is used to control the deviator stress. In a compression test the axial stress is thus the major principal stress σ_1; the intermediate and minor principal stresses (σ_2 and σ_3 respectively) are both equal to the cell pressure.

Connections to the ends of the sample permit either drainage of water or air from the voids of the soil or, alternatively, the measurement of pore pressure under conditions of no drainage.

In most standard tests the application of the allround pressure and of the deviator stress form two separate stages of the test; and tests are therefore classified according to the conditions of drainage obtaining during each stage:

(1) Undrained tests[i].—No drainage, and hence no dissipation of pore pressure, is permitted during the application of the all-round stress. No drainage is permitted during the application of the deviator stress (σ_1 - σ_3).

(2) Consolidated-undrained tests.—Drainage is permitted after the application of the all-round stress, so that the sample is fully consolidated under this stress. No drainage is permitted during the application of the deviator stress.

(3) Drained tests.—Drainage is permitted throughout the test, so that full consolidation occurs under the all-round stress and no excess pore pressure is set up during the application of the deviator stress.

In order to illustrate the inter-relation between results of the different types of test, saturated and partly saturated soils will be considered separately.

i. Alternative nomenclatures have been used both in Europe and the U.S.A. The present terms are considered to be the most descriptive of the test conditions.

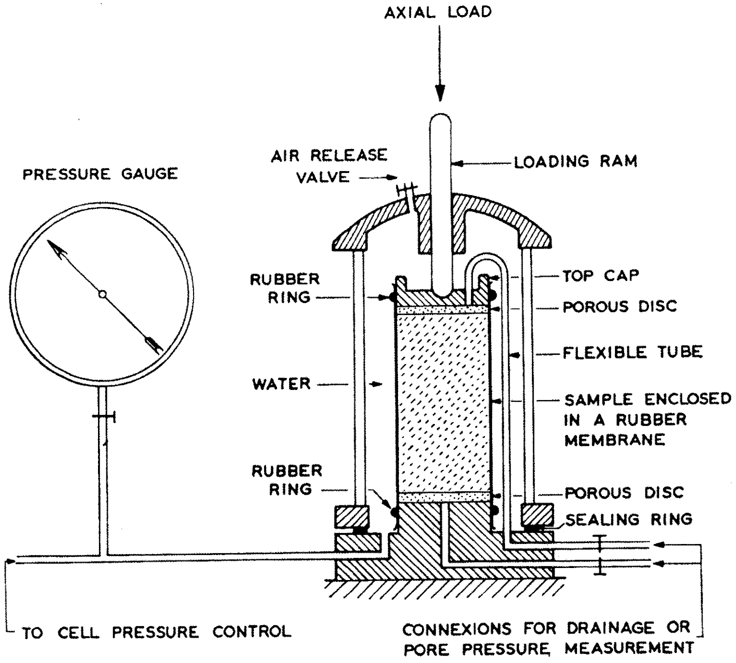

Fig. 5.—Diagrammatic layout of the triaxial cell.

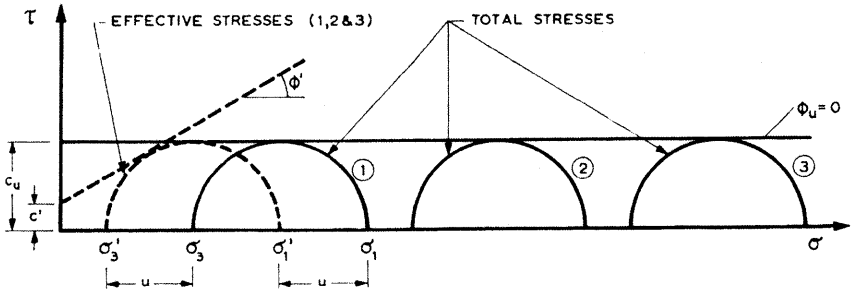

Fig. 6.—Undrained tests on saturated soil: total and effective stress circles.

(a) Undrained Tests on Saturated Cohesive Soils.—

These tests are carried out on undisturbed samples of clay, silt and peat as a measure of the existing strength of natural strata, and on remoulded samples when measuring sensitivity or carrying out model tests in the laboratory.

The compression strength (i.e. the deviator stress at failure) is found to be independent of the cell pressure, with the exception of fissured clays (discussed in section 6) and compact silts at low cell pressures. The corresponding Mohr stress circles are shown in Fig. 6.

If the shear strength is expressed as a function of total normal stress by Coulomb's empirical law:

$$\tau_f = c_u + \sigma \tan \Phi_u \tag{11}$$

where c_u denotes apparent cohesion,
Φ_u denotes angle of shearing resistance; in terms of total stress,
it follows that, in this particular case,

$$\left.\begin{aligned} \Phi_u &= 0 \\ c_u &= \frac{1}{2}(\sigma_1 - \sigma_3)_f \end{aligned}\right\} \tag{12}$$

The shear strength of the soil, expressed as the apparent cohesion, is used in a stability analysis carried out in terms of total stress, which, for this type of soil, is known as the $\Phi_u = 0$ analysis (Skempton, 1948 a and b). Since the value of c_u may be obtained directly from the unconfined compression test (where $\sigma_3 = 0$), and from the vane test in the field, it is a simple and economical test, but is often used without regard to the class of stability problem under consideration.

For fully saturated soils the increase in cell pressure is reflected in an equal increase in pore pressure and the effective stresses at failure remain unchanged. If pore pressure measurements are made during the test only one effective stress circle is obtained (Fig. 6), and tests at other water contents must be carried out to obtain the failure envelope in terms of effective stress.

In Fig. 7(a) an example is given of the changes in pore pressure during shear in an unconfined compression test and in Fig. 7(b) the Mohr circles are given in terms of total and effective stresses.

The A value measured in the undrained test on a sample of natural ground is very different from the value in situ under a similar change in shear stress. This results from the stress history given to the sample by changes in pore pressure which occur during sampling and preparation due to the removal of the insitu stresses, quite apart from disturbance due to the sampler itself. The release of the deviator stress existing in samples normally consolidated with no lateral yield is a major factor contributing to this effect.

Tests on samples anisotropically consolidated in the laboratory (Bishop and Henkel, 1953) and on undisturbed samples (Bishop, 1960) show that the effective stress in the sample when under an all-round pressure or unconfined can be less than half the effective overburden pressure in situ. Yet when the shear stress is increased to bring the sample to failure, the undrained strength closely corresponds to the in situ strength deduced from stability analysis or from vane tests. This is consistent with the experimental observation that, for a limited range of soil types and stress paths, strength

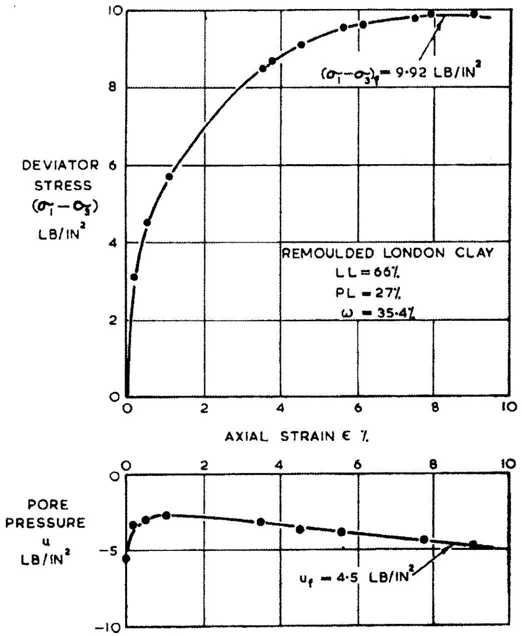

Fig. 7a.—Pore pressure change during shear in an unconfined compression test.

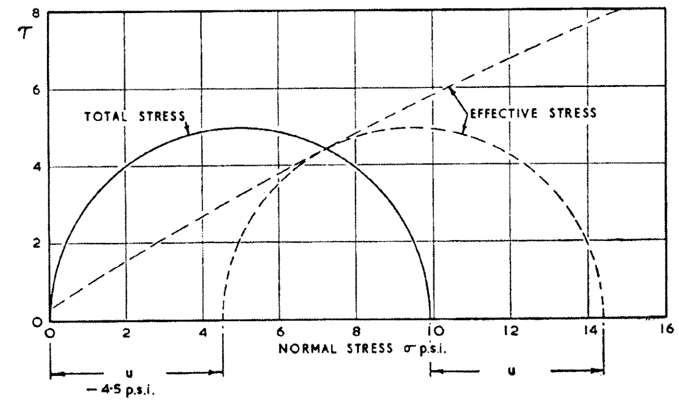

Fig. 7b.—Total and effective stress circles for the unconfined compression test.

and water content are uniquely related (Waterways Experiment Station, 1947; Henkel, 1959).

If it is indeed this fact which provides the empirical justification for the use of undrained compression tests in the $\Phi_u = 0$ analysis, then to reconsolidate the samples in the laboratory under the existing overburden pressure will inevitably lead to an overestimate of the in situ strength of the soil, since reconsolidation is almost always accompanied by a decrease in water content.

(b) Consolidated-Undrained Tests on Saturated Soils.—

These tests are carried out on both undisturbed and remoulded samples of cohesive soils, primarily to determine the values of c' and Φ', but also to determine the values of A and to study the effect of stress history.

In the standard test the sample is allowed to consolidate under a cell pressure of known magnitude (p), the three principal stresses thus being equal. The sample is then sheared under undrained conditions by applying an axial load. As in the case of the undrained test in the previous section, the cell pressure at which the sample is sheared does not influence the strength (except in dilatant silts at low pressures) as illustrated in Fig. 8e. The test result, in terms of total stresses, may thus be expressed by plotting the value of c_u against consolidation pressure p, Fig. 8b.

For normally consolidated soils the ratio c_u/p is found to be constant, its value depending on soil type. However, strengths measured in undrained triaxial tests and vane tests on strata existing in nature in a normally consolidated state, when plotted against the effective overburden pressure, lead to a lower estimate of c_u/p than is found with samples consolidated under equal all-round stress in the laboratory. The difference increases as the plasticity index decreases and appears to be due to two causes:

(i) A naturally deposited sediment is consolidated under conditions of no lateral displacement, and hence with a lateral effective stress considerably less than the vertical stress. The ratio of the effective stresses, termed the coefficient of earth pressure at rest, is generally found from laboratory tests to lie in the range 0.7-0.35, the lower values occurring in soils with a low plasticity index (Terzaghi, 1925; Bishop, 1958 a; Simons, 1958). This cause alone can account in soils of low plasticity for a difference of 50% in the value of c_u/p (for example, Bishop and Henkel, 1953; Bishop and Eldin, 1953).

(ii) Reconsolidation in the laboratory after the stress release associated with even the most careful sampling technique leads to a lower void ratio than would occur in the natural stratum under the same stress. The value of the pore pressure parameter A in particular is sensitive to the resulting modification in soil structure and this in turn leads to a higher undrained strength.

For these reasons the use of the results of consolidated-undrained tests, expressed in terms of total stress either by the parameter c_u/p or by the value of Φ_{cu} (see appendix 1), can be justified in few practical applications. However, if the pore pressure is measured during the undrained stage of the test, the results can be expressed in terms of the effective parameters c' and Φ'. Experience has shown that these parameters can be applied to a wider range of practical problems.

The relationships between the total stress, pore pressure and effective stress characteristics obtained in a typical series of consolidated-undrained

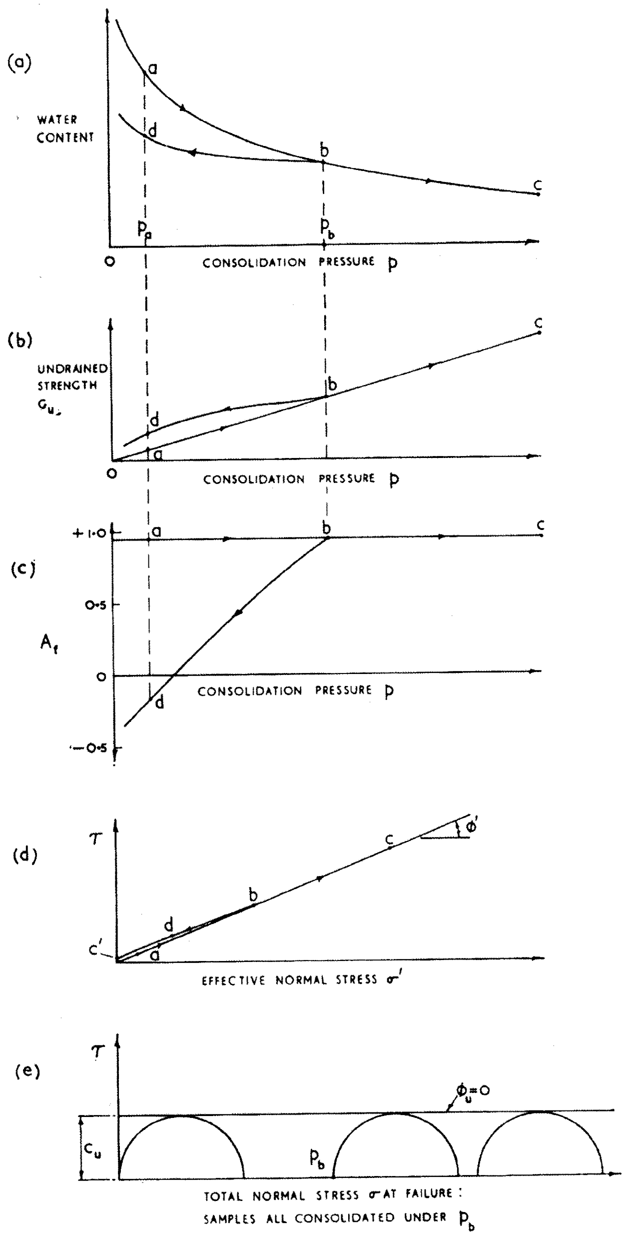

Fig. 8.—The relationships between the total stress, pore pressure and effective stress characteristics for a series of consolidated-undrained triaxial tests on saturated cohesive soil.

triaxial tests are illustrated in Fig. 8. The points, a, b, and c represent normally consolidated samples; the point d represents an over-consolidated sample, the overconsolidation ratio being p_b/p_d, Fig. 8a. For normally consolidated samples the effective stress envelope is a straight line with c' equal to zero (Fig. 8d), the value of Φ' depending on soil type. Over-consolidation results in an envelope lying a little above this straight line; the section of this envelope relevant to any particular practical problem can generally be represented with sufficient accuracy by a slightly modified value of Φ' and a cohesion intercept c'.

The most marked effect of over-consolidation is, however, on the value of A, which, with increasing over-consolidation ratio, drops from a value typically about 1 at failure to values in the negative range (Figs. 2 and 8c). These low A-values are, in turn, largely responsible for the high undrained strength values resulting from over-consolidation (compare point d with point a in Fig. 8b).

Values of c' and Φ' are usually based on the effective stress circles corresponding to maximum deviator stress. However, in some over-consolidated clays in which large decreases in pore pressure during shear are associated with very large failure strains, a slightly larger value of Φ' is obtained by plotting the state of stress at a smaller strain approximating to the point at which the ratio of the principal effective stresses σ_1'/σ_3' reached its maximum value. The difference in the value of Φ' is generally not important from a practical point of view, but in making comparisons between the values of Φ' obtained from consolidated-undrained and drained tests it is necessary to specify which definition of Φ' is being used[j].

(c) Drained Tests on Saturated Soils.—

Drained tests are carried out on both undisturbed and remoulded samples of cohesive soils to obtain directly the shear strength parameters relevant to the condition of long term stability, when the pore pressures have decreased (or increased) to their equilibrium values.

In the standard test the sample is allowed to consolidate under a cell pressure of magnitude p and is then sheared by increasing the axial load at a sufficiently slow rate to prevent any build-up of excess pore pressure. The effective minor principal stress σ_3' at failure is thus equal to p, the consolidation pressure; the major effective principal stress σ_1' is the axial stress. The test results lead directly to the effective stress shear parameters c' and Φ', which for drained tests are often denoted c_d and Φ_d.

The drained tests also provides data on the volume changes which occur during the application of the equal all-round stress and the deviator stress.

(d) Inter-Relationship between the three Types of Test on Saturated Soil.—

Two aspects of this inter-relationship are of practical interest to the engineer concerned with stability problems: (1) The degree of reliability with

j. Whether this difference reflects an actual characteristic of frictional materials or merely the increasing nonuniformity of stress in the cylindrical compression tests at large strains is still open to question. It is, however, clear from tests on sand published by the Waterways Experiment Station (1950) and other unpublished tests at the Norwegian Geotechnical Institute and at Imperial College that for very loose soil structures the maximum deviator stress may occur at smaller strains than the maximum stress ratio, and here the difference in Φ' (of up to 15° or so) undoubtedly represents a physical property of the soil.

which the effective stress envelope defined by the parameters c' and Φ' can be assumed to be the same for undrained, consolidated-undrained and drained tests; and (2) the extent to which volume changes in drained tests are an indication of the magnitude of pore pressure changes in consolidated-undrained tests.

In Fig. 9d are compared the results of undrained, consolidated-undrained, and drained tests on clay from the foundation of the Chew Stoke Dam (described by Skempton and Bishop, 1955). The close agreement between the effective stress failure envelopes may be noted. It is also of interest to note from the low values of A_f that an undisturbed sample reconsolidated in the laboratory behaves as though it were 'over-consolidated' even at cell pressures greatly in excess of the in situ pre-consolidation pressure. The intercept c' will in general not be zero for this part of the failure envelope.

That there should be close agreement between the effective stress envelopes for consolidated-undrained and drained tests on normally consolidated samples has been shown theoretically by Skempton and Bishop (1954) using the concept of true cohesion and friction due to Hvorslev (1937). Since Φ' is to some extent time-dependent, it is necessary to use similar rates of testing in making an experimental comparison, and to ensure adequate time for pore pressure measurement in the consolidated-undrained test and for drainage in the drained test. The predicted values of Φ' from the consolidated-undrained test are the higher, but only by 0-1° in typical cases[k].

However, for heavily over-consolidated clays the position is generally reversed[l] and the drained test is usually found to give the higher value, due to the work done by the increase in volume during shear in the drained test, and to the smaller strain at failure.

The volume changes in drained tests have for some time been known to correlate qualitatively with the pore pressure changes in undrained tests. Experimental data on two remoulded clays have recently been presented by Henkel (1959 and 1960) who has described a simple graphical procedure from which the quantitative relationship may be obtained.

(e) Undrained Tests on Partly Saturated Cohesive Soils.—

These tests are most commonly carried out on samples of earth-fill material compacted in the laboratory under specified conditions of water content and density. They are also applied to undisturbed samples of strata which are not fully saturated, and to samples cut from existing rolled fills and trial sections.

The compression strength is found to increase with cell pressure (Fig. 10a), as the compression of the air in the voids permits the effective stresses to increase. However, the increase in strength becomes progressively smaller as the air is compressed and passes into solution, and ceases when the stresses are large enough to cause full saturation, Φ_u the approximating to zero. The failure envelope expressed in terms of total stress is thus non-linear, and values of c_u and Φ_u can be quoted only for specific ranges of normal stress.

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k. The sign and magnitude of this difference may change if the failure strains are very dissimilar, as in long term tests reported by Bjerrum, Simons, and Torblaa (1958).

l. If the failure envelopes corresponding to maximum deviator stress are compared.

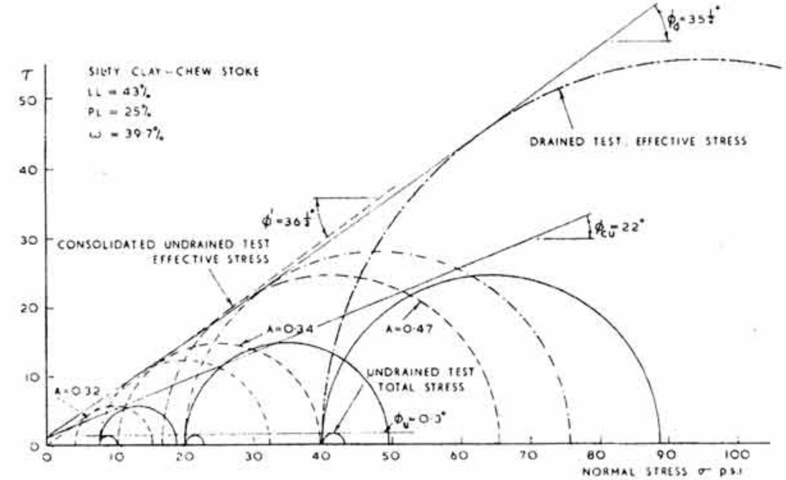

Fig. 9.—Undrained, consolidated-undrained and drained tests on undisturbed samples of Chew Stoke silty clay: maximum deviator stress.

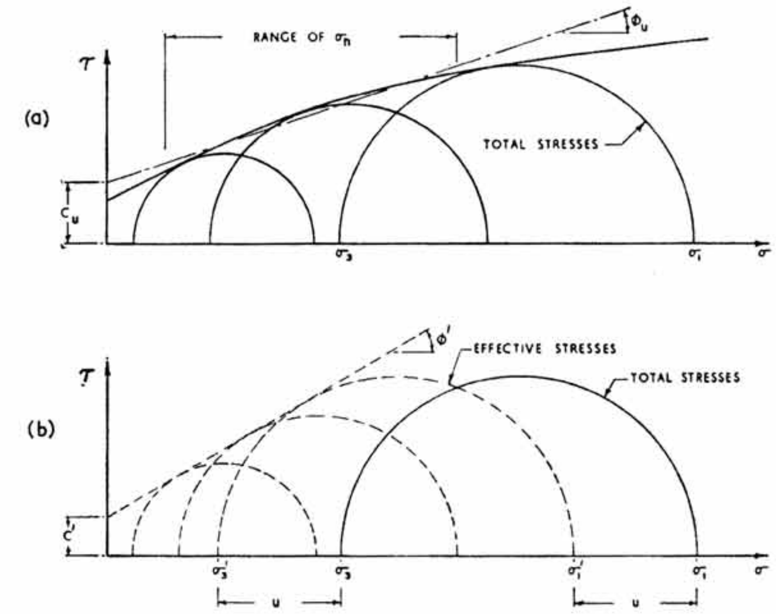

Fig. 10.—Undrained tests on partly saturated cohesive soil (a) in terms of total stress, (b) in terms of effective stress.

If the pore pressure is measured during the test, as is usual where field pore pressure measurements are to be used to check the stability during construction, then the failure envelope can be expressed in terms of effective stress, Fig. 10b. The effective stress envelope is found to approximate very closely to a straight line over a wide range of stress.

However, rather more difficulty arises in defining accurately the effective stress envelope for a partly saturated soil than at first apparent. The first difficulty lies in testing technique, to which attention was drawn by Hilf (1956). This problem is discussed in detail by Bishop (1960) and Bishop, Alpan, Blight, and Donald (1960), where it is concluded that accurate pore water pressure measurements can be made in the triaxial apparatus in partly saturated cohesive soils provided a porous element of very high air entry value is used and provided a considerably reduced rate of testing is accepted.

The second difficulty lies in the form of the expression for effective stress (equation 3), which includes a term for pore-air pressure as well as pore-water pressure for values of the factor x other than unity. The use of the simple expression for effective stress of total stress minus porewater pressure leads to an over-estimate of effective stress of $(1 - x)(u_1 - u_2)$ where $(u_1 - u_2)$ is the difference between pore-air pressure and pore water pressure. Since values of $(u_1 - u_2)$ of up to 40 lb. per sq. inch have already been measured on rolled fill in the triaxial test, and the value of x approximates to the degree of saturation, significant errors in effective stress result from the use of the simpler expression. This is particularly marked near the origin of the Mohr diagram and may lead to the apparent anomaly of a negative 'cohesion' intercept (Bishop, Alpan, Blight, and Donald, 1960). However, pore pressures set up under construction conditions are only critical if the water content of the fill and the magnitude of the stresses lead to almost full saturation, and in this case the error is small enough to be ignored in many practical problems.

(f) Consolidated-Undrained Tests on Partly Saturated Cohesive Soils.—

These tests are carried out on samples of compacted earth-fill material and on undisturbed samples. They may be necessary to determine c' and ϕ' when the degree of saturation of the samples is not low enough to result in a sufficient range of strengths in the undrained tests to define a satisfactory failure envelope.

Consolidated-undrained tests in which a backpressure is applied to the pore space to ensure full saturation before shearing are carried out to examine the effect on the values of c' and ϕ' of the submergence of fill or foundation strata. Back-pressures of up to 100 lb. per sq. inch are often required to give full saturation on a short term basis.

(g) Drained Tests on Partly Saturated Cohesive Soils.—

Drained tests are carried out on both compacted and undisturbed samples to obtain directly the values of c' and ϕ' for the condition of long term stability. Generally a backpressure is applied to ensure full saturation of the sample before the application of the deviator stress, during which the backpressure is held constant.

(h) Inter-Relationship between the Three Types of Test on Partly Saturated Soil.—

Here again two aspects of this inter-relationship are of practical interest to the engineer concerned with stability problems: (1) The comparison of the values of c' and ϕ' obtained from the different types of test; and (2) the prediction of pore pressure changes from volume changes.

Tests carried out at Imperial College have generally shown that the difference between the values of ϕ' measured in the different types of test are not very significant from a practical point of view. The value of c', however, tends to correlate with water-content at failure. Where all the samples defining a failure envelope show a marked increase in water content in the consolidated-undrained or drained test with a back-pressure, c' is generally reduced. With the lower values of c' obtained by using the improved pore pressure technique described elsewhere (Bishop, 1960; Bishop, Alpan, Blight, and Donald, 1960), the difference in c' obtained in the different tests are less marked, and in some soils are not of practical significance[m] (Fig. 11). The range of soil types so far tested using this technique is, however, rather limited.

It is generally easier to make accurate measurements of pore water pressure under undrained conditions than to make the necessarily very accurate measurements of volume change and degree of saturation on which pore pressure predictions depend. Studies at the Bureau of Reclamation by Bruggeman et al. (1939), Hamilton (1939), Hilf (1948 and 1956) have shown that the change in pore-air pressure can be related to observed volume changes by the use of Boyle's law and Henry's law. However, the magnitude of the difference between pore-air and porewater pressure still has to be found experimentally. For practical purposes, where the pore water pressure is the more significant factor, it is therefore more convenient to measure it directly, particularly if the effect of stress ratio on pore pressure is also to be studied[n].

(i) Advantages and Limitations of the Triaxial Test.—

The advantages and limitations of the triaxial test have been discussed in some detail elsewhere (Bishop and Henkel, 1957), and will be referred to only briefly here.

The principal advantages of the triaxial test as performed on cylindrical specimens are that it combines control of the drainage conditions and the possibility of the measurement of pore pressure with relative simplicity in operation.

The principal limitations are that the intermediate principal stress cannot be varied to simulate plane strain conditions, that the directions of the principal stresses cannot be progressively changed, and that end restraint may modify the various relationships between stress, strain, volume change, and pore pressure.

For most practical purposes the advantages outweigh the limitations, and it will be apparent from section 6 that a very satisfactory correlation does in fact exist between laboratory tests and field observations of stability in many important engineering problems.

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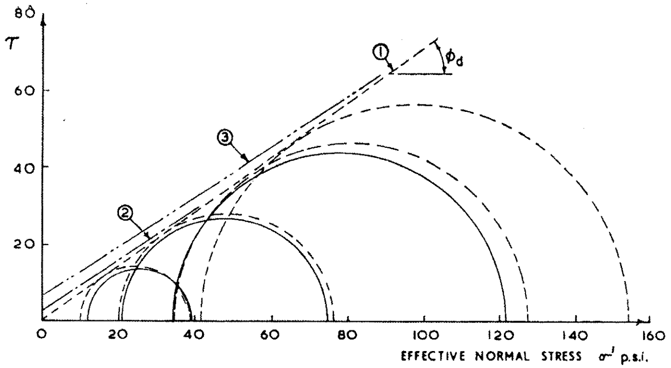
m. Some difference will in general arise from such factors as the different strains at which 'failure' is taken to occur, the different rates of volume change at failure, and, in soils having true cohesion in the Hvorslev sense, the different water contents of the samples defining the failure envelope.

n. The effect of stress ratio is discussed by Bishop (1952 and 1954 a) and Fraser (1957).



STRESS CIRCLES FOR DRAINED TESTS WITH FULL SATURATION SHOWN BY BROKEN LINE
STRESS CIRCLES FOR CONSOLIDATED UNDRAINED TESTS WITH FULL SATURATION, PLOTTED IN TERMS OF EFFECTIVE STRESS AT MAX. DEVIATOR STRESS, SHOWN BY SOLID LINE

ENVELOPE (1) REPRESENTS DRAINED TESTS
ENVELOPE (2) REPRESENTS UNDRAINED TESTS IN TERMS OF σ-u (CIRCLES NOT SHOWN)
ENVELOPE (3) REPRESENTS UNDRAINED TESTS IN TERMS OF EQN (3) WITH ASSUMED X-VALUES

STRAIN RATE 0·38% PER HOUR IN ALL TESTS

Fig. 11.—Undrained, consolidated-undrained and drained tests on boulder clay compacted at an initial water content 2% dry of optimum: clay fraction 4%.

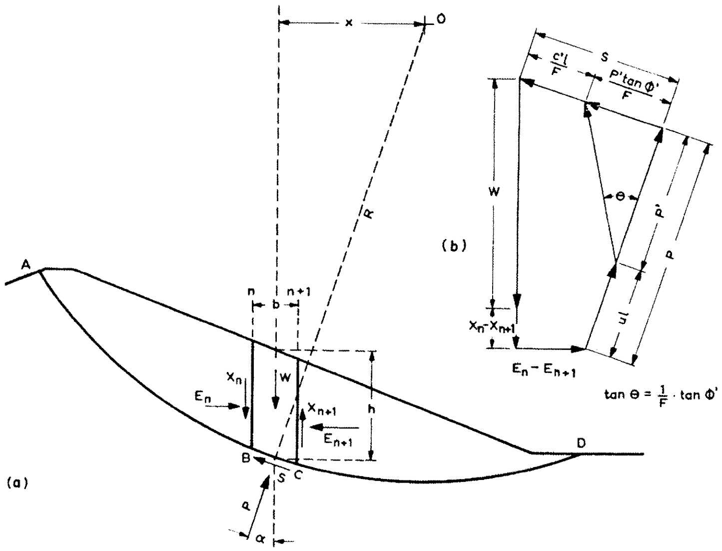


Fig. 12.—Forces in the slices method of stability analysis.

5. METHODS OF STABILITY ANALYSIS

The stability of soil masses against failure under their own weight, or under the action of applied loads, can be examined either by methods based on elastic theory or by methods based on the principle of limit design.

In the first case the stress distribution is calculated and the maximum stresses are then compared with the strength of the soil. As a practical method it is, however, open to several serious objections. Firstly, it is difficult to assess the error resulting from the assumption that the soil mass is a homogeneous elastic material having elastic constants which are independent of the magnitude of the stresses. Secondly, it has been shown that, even if these assumptions were true, local overstress would occur in a typical earth dam section when its factor of safety (by a slip circle method) lay below a value of about 1.8 (Bishop, 1952). The same applies in principle to earth slopes and foundations.

In consequence elastic methods are not applicable to the calculation of the factor of safety when studying observed failures or for design work on embankments and cuts where 1.5 is accepted as a working value for factor of safety. Elastic methods are, however, useful in giving an estimate of the stress distribution studies and for pore pressure prediction.

In most practical stability problems, therefore, the engineer is concerned with the factor of safety against complete failure, rather than against local overstress. The most general definition of factor of safety against complete failure, which can be applied irrespective of the shape of the failure surface, is expressed in terms of the proportion of the measured shear strength that must be mobilized to just maintain limiting equilibrium. The shear strength parameters to which the factor of safety is applied in setting up the equations expressing the condition of limiting equilibrium depend on whether the analysis is carried out in terms of effective stress (c', Φ' analysis) or total stress ($\Phi_u = 0$ analysis). The two cases will be treated separately.

(a) Effective Stress Analysis.—

In the effective stress analysis the proportion of the shear strength mobilized for limiting equilibrium is expressed:

$$\tau = (c'/F) + (\sigma - u)(\tan\Phi'/F) \tag{13}$$

The value of the factor of safety F is obtained by assuming limiting equilibrium along a trial slip surface (usually the arc of a circle in cross-section), balancing the forces and solving for F. The value of σ is determined from the equilibrium of the soil mass above the failure surface by an appropriate graphical or numerical method. The method of determining the value of u will depend on the class of stability problem.

(I) In class (a) problems, where the pore pressure is an independent variable, the value of u will be obtained from ground water level if there is no flow, or from a flow net if a state of steady seepage exists. The flow net can either be calculated or based on field measurements of pore pressure.

(II) In class (b) problems, where the magnitude of the pore pressure depends on the stress changes tending to lead to instability, the most practical method of approach is that adopted in earth dam design. Here a prediction is made of the actual pore pressure likely to obtain in the stable dam, which should thus check with the field pore pressure measurements usually made during construction. This prediction is based on an approximate stress distribution within the dam, the undrained pore pressure parameter $\overline{B}$ and a calculated allowance for pore pressure dissipation, the value of $\overline{B}$ being re-adjusted if necessary to match the calculated factor of safety.

Where field measurements of pore pressure are available they are of course substituted directly in the analysis.

While any method of stability analysis can be used which correctly represents the statics of the problem, the more complex soil profiles or dam sections involving a number of zones of c' and Φ' and irregular distributions of pore pressure can be handled most readily by a numerical form of the method of slices (Bishop, 1954 b).

As applied to the slip circle analysis (Fig. 12), the method leads to an expression for the factor of safety:

$$F = \frac{1}{\Sigma W \sin \alpha} \cdot \sum \left[\{c'b + \tan \Phi' . [W(1 - r_u) + (X_n - X_{n+1})]\} \frac{\sec \alpha}{1 + \frac{\tan \Phi' . \tan \alpha}{F}} \right] \quad (14)$$

This expression takes full account of both horizontal and vertical forces between the slices. The vertical shear force term, which cannot be eliminated mathematically, can however be put equal to zero with little loss in accuracy. The method agrees to within about 1% with the modified friction circle method described by Taylor (1948) in two cases which have been checked.

The programming of the digital electronic computer 'DEUCE' for the numerical method by Little and Price (1958) has given it an additional and overwhelming advantage, since any specified pattern of slip circles can be analysed at a rate of about 5 seconds per circle using about 30-50 slices. This leaves the engineer free to investigate the effect of varying his assumptions about soil properties and pore pressure, and to modify his design, without the heavy burden of computation previously involved.

The extension of the slices method to noncircular surfaces has been undertaken by Janbu (1954 and 1957) and Kenney (1956) and it is at present being programmed for the computer.

(b) Total Stress Analysis.—

In the total stress analysis the proportion of the shear strength mobilized is expressed, for the $\Phi_u = 0$ condition, as:

$$\tau = c_u / F \quad (15)$$

In the notation of Fig. 12, the expression for the factor of safety using the slip circle analysis becomes:

$$F = \Sigma c_u l / \Sigma W \sin \alpha \quad (16)$$

When $\Phi_u = 0$ the inter-slice forces enter into the calculation only if a non-circular slip surface is used.

For saturated soils the apparent cohesion c_u is equal to one half of the undrained compression strength (equ. 12) and its value is obtained from undrained tests on undisturbed samples or from vane tests. The value of c_u usually varies with depth and appropriate values must be used around the trial failure surface.

It should be noted that the use of this method is correct only where the field conditions correspond to the laboratory tests conditions, i.e. where the shear stress tending to cause failure is applied under undrained conditions[o]. It cannot in general be applied using undisturbed samples from slopes, for example, where the water content has had time to adjust itself to the stress changes set up by the formation of the slope.

The validity of the $\Phi_u = 0$ method is in fact restricted to saturated soils[p] and to problems in which insufficient time has elapsed after the stress change considered for an increase or decrease in water content to occur. It is therefore an 'end of construction method'. Whether the factor of safety subsequent to construction will have a lower value depends on the sign and magnitude of the stress changes. The particular cases are discussed in Section 6.

The use of total stress methods in which Φ_u is not zero, or in which the angle of consolidated undrained shearing resistance Φ_{cu} is used, is, in the opinion of the authors, to be avoided except in special cases, owing to the difficulty of determining the physical significance of the factor of safety thus obtained.

(c) Relationship between Total and Effective Stress Methods of Stability Analysis.—

Since the failure criterion and the associated method of stability are only convenient means of linking the stability problem with the appropriate laboratory test, a soil mass in limiting equilibrium should be found to have a factor of safety of 1 by whichever method the analysis is performed. As total stress methods can only be applied under undrained conditions, it is convenient to demonstrate this point by a simplified analysis of a vertical cut in saturated clay immediately after construction (Fig. 13).

To simplify the mathematics of the problem it is assumed that the undrained strength c_u does not vary with depth, and that the effective stress failure envelope is represented by c' = 0 and a constant value of Φ'. The failure surface is assumed to approximate to a plane without tension cracks.

The critical height H under these conditions is known to be equal to $4c_u/\gamma$ where γ is the density of the soil. The factor of safety of the soil adjacent to a vertical cut of depth H can be calculated in terms of either total or effective stress:

(I) Total Stress.—From equation 16:

$$\begin{aligned} F &= (\Sigma c_u . l) / (\Sigma W \sin \alpha) \\ &= (c_u . H \operatorname{cosec} \alpha) / (^1/_2 . \gamma H^2 . \cot \alpha \sin \alpha) \\ &= (2c_u) / (\gamma . H \cos \alpha \sin \alpha) \end{aligned} \quad (17)$$

Putting $dF/d\alpha = 0$ to obtain the value of α giving the lowest value of F we obtain $\alpha = 45^\circ$.

Substituting in equ. 17 gives:

$$F = 4c_u / \gamma H \quad (18)$$

Substituting $4c_u/\gamma$ for H we obtain F = 1.

o. The error introduced by the fact that the principal stress directions in most practical problems differ from those in the laboratory is discussed by Hansen and Gibson (1949).

p. Stiff fissured clays under a reduction of normal stress are an exception.

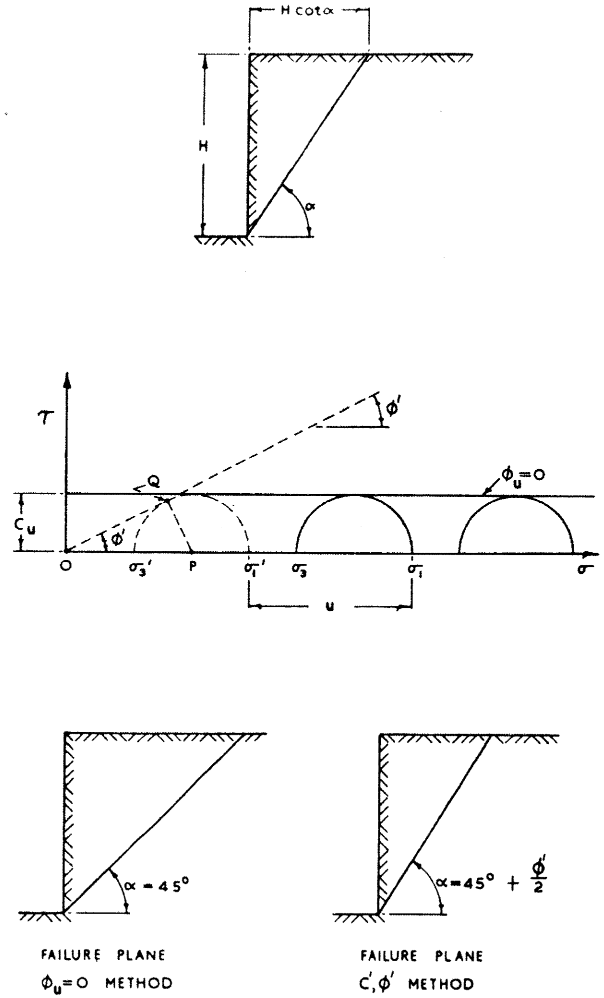


Fig. 13.—Simplified analysis of the stability of a vertical cut in saturated cohesive soil immediately after excavation, using both total and effective stress methods.

(II) Effective Stress.—From the Mohr diagram in Fig. 13 we can obtain the pore pressure in an element of soil at failure in terms of the major principal stress. It follows from the geometry of the triangle OPQ that:

$$(\sigma_1 - \sigma_3)/2 = [(\sigma_1 - u) - \{(\sigma_1 - \sigma_3)/2\}] \sin \Phi' \tag{19}$$

Putting $(\sigma_1 - \sigma_3)/2 = c_u$ and rearranging we obtain:

$$u = \sigma_1 - c_u \,.\, [(1 + \sin \Phi')/(\sin \Phi')] \tag{20}$$

For a plane slip surface, and with $c' = 0$ the expression for F given in equ. 15 simplifies to the form:

$$F = [1/(\Sigma W \sin \alpha)] \,.\, \Sigma[\tan \Phi'(W \cos \alpha - ul)] \tag{21}$$

For failure on a plane, the state of stress corresponds in this case to the Rankine active state, and thus the major principal stress σ_1 is equal to γh, the vertical head of soil above the element.

Substituting in equ. 21 the value of u given by equ. 20, and putting $c_u = \gamma H/4$, we obtain the expression for F:

$$F = \tan \Phi' (\cot \alpha - [1/\sin 2\alpha] \,.\, [(\sin \Phi - 1)/(\sin \Phi)]) \tag{22}$$

Putting $dF/d\alpha = 0$ we now find that the minimum value of F is given by the inclination $\alpha = 45^0 + \Phi'/2$. Substituting this value in equation 22 and expressing the angles in terms of $\Phi'/2$, we find that the expression again reduces to F = 1.

This comparison illustrates two important conclusions. Firstly, both total and effective stress methods of stability analysis will agree in giving a factor of safety of 1 for a soil mass brought into limiting equilibrium by a change in stress under undrained conditions. Secondly, although the values of factor of safety are the same, the position of the rupture surface is found to depend on the value of Φ used in the analysis. The closer this value approximates to the true angle of internal friction, the more realistic is the position of the failure surface[q], and this is confirmed by the analysis of the Lodalen slide in terms of c' and Φ' (Sevaldson, 1956 and Section 6).

The choice of method in short term stability problems in saturated soil is thus a matter of practical convenience and the $\Phi_u = 0$ method is generally used because of its simplicity, unless field measurements of pore pressure are to be used as a control. It should be noted, however, that for factors of safety other than 1 the two methods will not in general give numerically equal values of F. In the effective stress method the pore pressure is predicted for the stresses in the soil, under the actual loading conditions, and the value of F expresses the proportion of c' and tan Φ' then necessary for equilibrium. The total stress method on the other hand implicitly uses a value of pore pressure related to the pore pressure at failure in the undrained test. The high factor of safety shown for example in the $\Phi_u = 0$ analysis of a slope of over-consolidated clay in which the pore pressure shows a marked drop during the latter stages of shear will therefore not be reflected in the effective stress analysis in such a marked way.

It cannot be too strongly emphasized that a comparison between effective stress and total stress methods can only be logically made when the shear stress tending to cause instability has been applied under undrained conditions. The use of the $\Phi_u = 0$ method under other conditions cannot be justified theoretically and in practice often leads to very unrealistic results (see Table V).

q. This point is discussed more fully by Terzaghi, 1936 b; Skempton, 1948 a; and Bishop, 1952.

6. THE APPLICATION OF STABILITY ANALYSIS TO PRACTICAL PROBLEMS

In this section the stability analysis of a number of typical engineering problems will be examined. The purpose of the examination is in the first place to obtain a clear qualitative picture of what happens to the variables controlling stability during and after the construction operation or load change under consideration. The second purpose of the examination is to indicate the most dangerous stage from the stability point of view and to select the appropriate shear parameters and method of stability analysis.

It is not possible to generalise about the solution of practical problems without considering the principal properties of the soil in each case. It will have been apparent from section 2 that the permeability of the soil has an important bearing on the way in which the stability problem is treated. In the more permeable soils (e.g. sands and gravels) the pore pressure will be influenced by the magnitude of the stresses tending to lead to instability only under conditions of transient loading. Both end of construction and long term problems will fall into class (a) in which pore pressure is an independent variable. Only in the less permeable soils do the relative merits of alternative methods of analysis have to be considered in most practical cases.

In Table I are listed representative values of the shear strength parameters of some typical soils arranged in order of decreasing permeability. The wide range of permeability values will be noted, and it will be apparent that it is here that the largest quantitative difference between the soil types lies.

Table I.—Permeability and Shear Strength Parameters of Typical Soils. (* Signifies Undisturbed Samples).

Material	Plasticity Index P I %	Permeability K cm/sec. (Approx.)	c' lb./sq.ft.	Φ' Degrees
Rock fill: tunnel spoil	–	5	0	45
Alluvial gravel: Thames Valley	–	5×10^{-2}	0	43
Medium sand: Brasted	–		0	33
Fine sand	–	1×10^{-4}	0	20–35
Silt: Braehead	–	3×10^{-5}	0	32
Normally consolidated clay of low plasticity – Chew Stoke*	20	1.5×10^{-8}	0	32
Normally consolidated clay of high plasticity – Shellhaven*	87	1×10^{-8}	0	23
Over-consolidated clay of low plasticity – Selset boulder clay*	13	1×10^{-8}	170	$32^1/_2$
Over-consolidated clay of high plasticity – London clay*	50	5×10^{-9}	250	20
Quick clay*	5	1×10^{-8}	0	10–20

The more important problems are:

(a) Bearing Capacity of a Clay Foundation.—

This problem may be illustrated most simply in terms of the construction of a low embankment on a saturated soft clay stratum with a horizontal surface. In Fig. 14a is shown diagrammatically the variation with time of the

factors which govern stability, i.e. the average shear stress along a potential sliding surface and the average pore pressure ratio.

The excess pore pressure set up in an element of clay beneath the embankment is given by the expression:

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)] \tag{4}$$

For points beneath the embankment Δu will in general be positive and have its greatest value at the end of construction, since B = 1 and A is positive for normally or lightly overconsolidated clay. Unless construction is slow or the clay contains permeable layers, little dissipation of pore pressure will occur during the construction period. After construction is completed the average value of r_u will decrease as redistribution and dissipation of the excess pore pressures occur, until finally the pore pressures correspond to ground water level.

The factor of safety given by the effective stress analysis will thus show a minimum value at or near the end of construction, after which it will rise to the long term equilibrium value[r]. For the long term stability calculation it is obviously appropriate to take the values of c' and Φ' from drained tests. For the end of construction case the same values may also be used, for, though it is more logical to take the value from undrained and consolidated-undrained tests expressed in terms of effective stress, the error is on the conservative side and is likely to be small.

The use of the effective stress method for the end of construction case means, however, that the pore pressures must be predicted or measured in the field. Typical field measurements of pore pressure under an oil storage tank are illustrated in Fig. 14b (after Gibson and Marsland, 1960). However, field measurements are usually limited to the more important structures and to earth dams, and the application of this method to the end of construction case will in other instances have to depend on estimated pore pressure values. As this estimate involves an assumption about the stress distribution (which is influenced by how nearly limiting equilibrium is approached) and the determination of the value of A, it is usually avoided by going directly to the $\Phi_u = 0$ analysis which is applicable to the end of construction case with zero drainage.

The undrained shear strength to be used in the $\Phi_u = 0$ analysis is obtained from undrained triaxial tests (or unconfined compression tests) on undisturbed samples, or from vane tests in the field. In the majority of problems involving foundations on soft clay, where it is quite clear that the long term factor of safety is higher than the value at the end of construction, there is then no need for the more elaborate testing and analysis required by the effective stress method. However, if appreciable dissipation of pore pressure is likely to occur during construction, it is uneconomical not to take advantage of it in calculating the factor of safety and the effective stress method is then required.

The failure of a bauxite dump at Newport (reported by Skempton and Golder, 1948), may be taken as an example of the use of the $\Phi_u = 0$ analysis for end of construction conditions (Fig. 15). After relatively rapid tipping,

r. The position of the most critical slip surface will of course change as the pressure pattern alters.

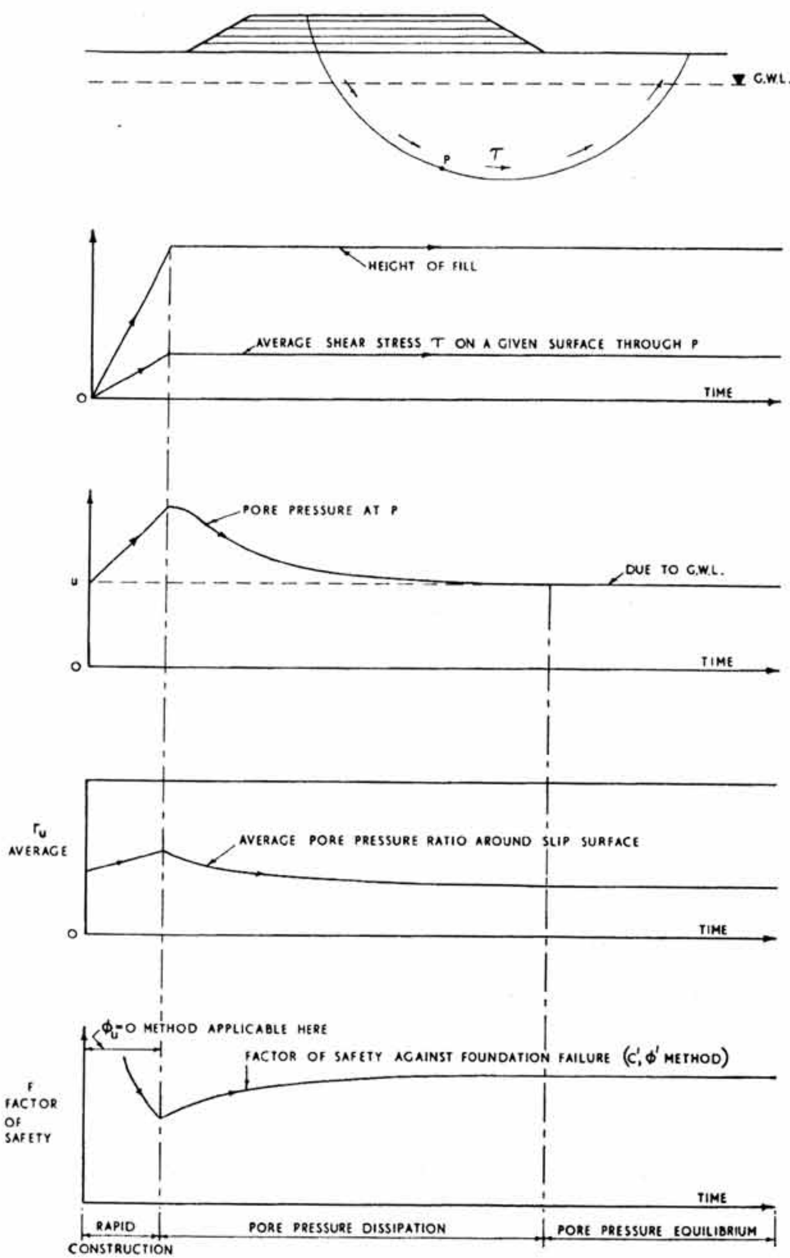


Fig. 14a.—Variation with time of the shear stress, local and average pore pressure, and factor of safety for the saturated clay foundation beneath a fill.

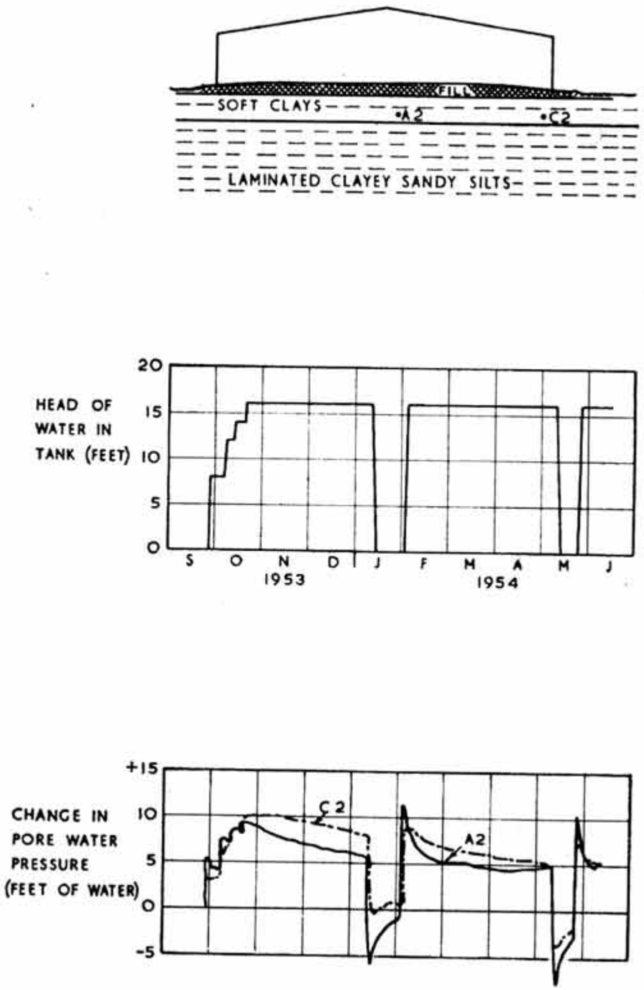


Fig. 14b.—Pore pressure changes in a soft clay foundation on filling and emptying a storage tank (after Gibson and Marsland, 1960).

failure occurred at a height of 25 feet; the factor of safety by the $\Phi_u = 0$ analysis was subsequently found to be 1.08, which can be accepted as agreement to within the limit of experimental accuracy.

In this case the fill was a granular material and its contribution to the shearing resistance was small. In cases where the fill is a cohesive material of high undrained strength, the use of the full value of this strength in the $\Phi_u = 0$ analysis gives misleading results. The explanation appears to be that shear deformations set up in the soft clay foundation under undrained loading

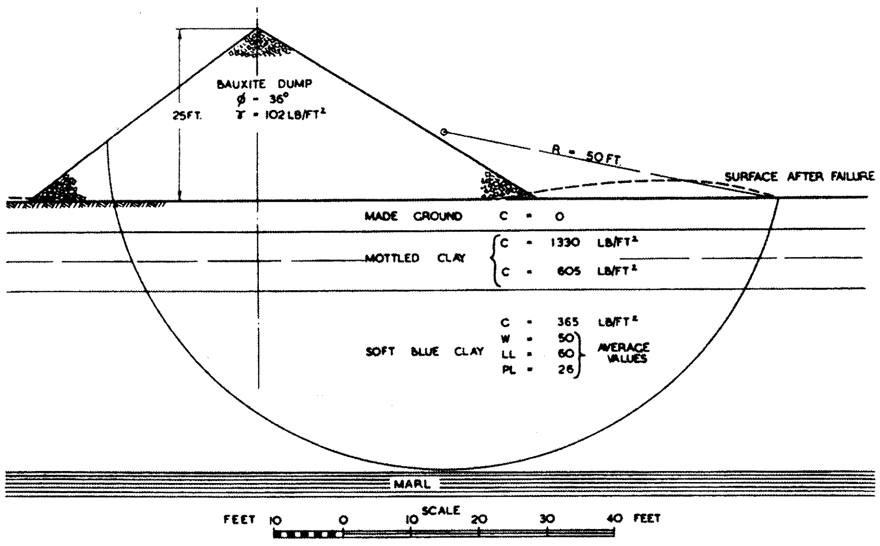


Fig. 15.—Failure of a bauxite dump at Newport (after Skempton and Golder, 1948).

set up tensile stresses in horizontal direction in the more rigid fill above and result in vertical cracks. A description of cracks wider at the bottom than at the top and passing right through the fill is given by Toms (1953a).

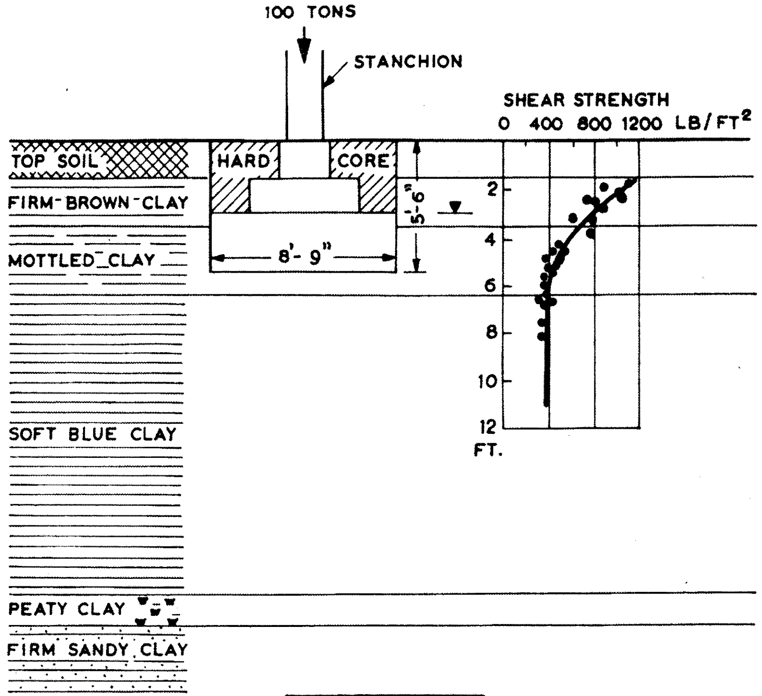
As the factor of safety rises with time, no long term failures can be quoted in this category.

The calculation of the ultimate bearing capacity of a structural foundation on a saturated clay is in principle the same as the problem treated above. However, the bearing capacity is not calculated by assuming a circular sliding surface, but is computed from the theory of plasticity for both the total and effective stress analyses, the results being expressed directly as bearing capacity factors.

For large foundations on soft clay the ultimate bearing capacity will increase with time after loading. For small, shallow foundations on stiff clay the ultimate bearing capacity will decrease with time, but in most cases settlement considerations will govern the design.

An example of the $\Phi_u = 0$ analysis of an end of construction foundation failure has been given by Skempton (1942). Here a footing 8 feet by 9 feet founded on a soft clay with c_u = 350 lb./sq. ft. failed at a nett foundation pressure of 2500 lb./sq. ft. (Fig. 16). Using a bearing capacity factor of 6.7 for this depth to breadth ratio, a factor of safety of 0.95 is obtained.

Two series of loading tests on the stiff fissured London clay may also be mentioned, where the $\Phi_u = 0$ analysis has led to factors of safety about 1.02 (Skempton 1959). These tests are particularly interesting as showing that the effect of fissures, which can lead to serious difficulties with end of construction problems in open excavations (see section 6b) does not prevent the successful use of the $\Phi_u = 0$ analysis in bearing capacity calculations under end of construction conditions.



W	LL	PL
40	70	29
40-60	70	28
60	70	27

NETT FOUNDATION PRESSURE AT FAILURE = 2500 LB/FT²
IF q = 6·7 c FACTOR OF SAFETY = 0·95

Fig. 16.—Failure of a foundation on soft clay at Kippen (after Skempton, 1942).

An example of the long term failure of a small heavily loaded foundation of stiff clay is more difficult to find. The long term failure of tunnel arch footings described by Campion (1951) probably falls in this category.

It may however be concluded that with few exceptions the end of construction conditions is the most critical for the stability of foundations and that for saturated clays this may be examined more simply by the $\Phi_u = 0$ analysis. From the field tests and full scale failures tabulated in Table II it is apparent that an accuracy of $\pm$ 15% can be expected in the estimate of factor of safety. One of the exceptions is dealt with in section 6(i). Where partial dissipation of pore pressure occurs during construction an analysis in terms of effective stress is used, and examples of the analysis are discussed in section 6(h).

Table II.—End of Construction Failures of Footings and Fills on a Saturated Clay Foundation: $\Phi_u = 0$ Analysis.

1. Footings, loading tests.

Locality	Data of clay					Safety factor $\Phi_u = 0$ analysis	Reference
	W	LL	PL	PI	$\frac{W-PL}{PI}$		
Loading test, Marmorerá	10	35	15	20	-0.25	0.92	Haefeli, Bjerrum
Kensal Green	–	–	–	–	–	1.02	Skempton 1959
Silo, Transcona	50	110	30	80	0.25	1.09	Peck, Bryant 1953
Kippen	50	70	28	42	0.52	0.95	Skempton 1942
Screw pile, Lock Ryan	–	–	–	–	–	1.05	Morgan 1944, Skempton 1950
Screw pile, Newport	–	–	–	–	–	1.07	Wilson 1950
Oil tank, Fredrikstad	45	55	25	30	0.67	1.08	Bjerrum, Øverland 1957
Oil tank A, Shellhaven	70	87	25	62	0.73	1.03	Nixon 1949
Oil tank B, Shellhaven	–	–	–	–	–	1.05	Nixon (Skempton 1951)
Silo, U.S.A.	40	–	–	–	–	0.98	Tschebotarioff 1951
Loading test, Moss	9	–	–	–	–	1.10	NGI
Loading test, Hagalund	68	55	20	35	1.37	0.93	Odenstad 1949
Loading test, Torp	27	24	16	8	1.39	0.96	Bjerrum 1954 c
Loading test, Rygge	45	37	19	18	1.44	0.95	Bjerrum 1954 c

2. Fillings

Chingford	90	145	36	109	0.50	1.05	Skempton, Golder 1948
Gosport	56	80	30	50	0.48	0.93	Skempton 1948 d
Panama 2	80	111	45	66	0.53	0.93	Berger 1951
Panama 3	110	125	75	50	0.70	0.98	Berger 1951
Newport	50	60	26	34	0.71	1.08	Skempton, Golder 1948
Bromma II	100	–	–	–	1.00	1.03	Cadling, Odenstad 1950
Bocksjön	100	90	30	60	1.17	1.10	Cadling, Odenstad 1950
Huntington	400	–	–	–	–	0.98	Berger 1951

Table III.—End of Construction Failures in Excavations: $\Phi_u = 0$ Analysis.

Location	Soil type	Data of Clay					Factor of safety: $\Phi_u = 0$ analysis	Reference
		W	LL	PL	PI	$\frac{W-PL}{PI}$		
Huntspill	Intact clay	56	75	28	47	0.6	0.90	Skempton, Golder, 1948
Congress Street	Intact clay	24	33	18	15	0.4	1.10	Ireland, 1954
Skattsmanso I	Intact clay	101	98	39	59	1.05	1.06	Cadling, Odenstad, 1950
Skattmanso II	Intact clay	73	69	24	45	1.09	1.03	Cadling, Odenstad, 1950
Bradwell	Stiff-fissured clay	33	95	32	63	0.02	1.7	Imperial College, 1959 (Skempton, La Rochelle)

(b) The Stability of Cuts and Free-Standing Excavations in Clay.—

The changes in pore pressure and factor of safety during and after the excavation of a cut in clay are illustrated in Fig. 17.

The change in pore pressure can conveniently be expressed by putting B = 1 and re-arranging equ. (4) in the form:

$$\Delta u = [(\Delta\sigma_1 + \Delta\sigma_3)/2] + (A - 1/2)(\Delta\sigma_1 - \Delta\sigma_3) \tag{23}$$

The reduction in mean principal stress will thus lead to a decrease in pore pressure, and the shear stress term will also lead to a decrease in pore pressure unless A is greater than 1/2, if the unknown effect on pore pressure of changing the directions of the principal stresses is neglected. An estimate of the stress distribution can be made from elastic theory if the initial factor of safety of the slope is high, or from the state of limiting equilibrium round a potential slip surface if the factor of safety is close to 1.

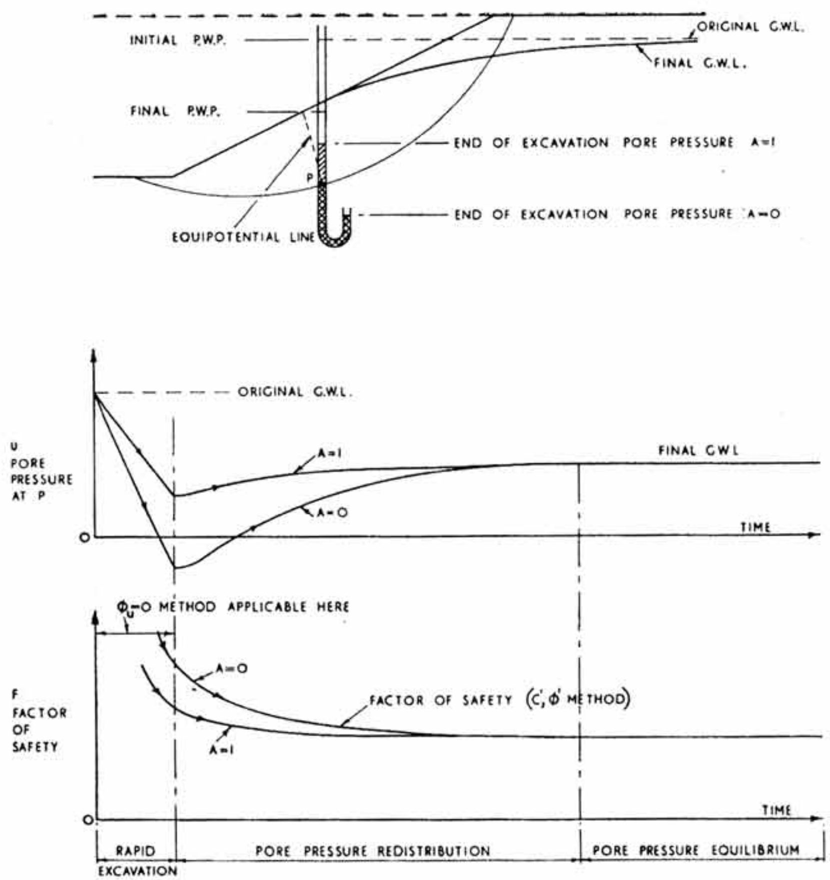


Fig. 17.—The changes in pore pressure and factor of safety during and after the excavation of a cut in clay.

In Fig. 17 the changes in pore pressure at a representative point are shown for the values A = 1 and A = 0. The final equilibrium values of pore pressure are taken from the flow pattern corresponding to steady seepage[s].

Using values of c' and ϕ' from drained tests or consolidated-undrained tests expressed in terms of effective stress the factor of safety can be calculated at all stages from equation 14. In the majority of cases, unless special drainage measures are taken to lower the final ground water level, the factor of safety reaches its minimum value under the long term equilibrium pore pressure conditions.

An example of the investigation of a long term failure of a cut in terms of effective stress has been given by Sevaldson (1956). The slide took place in 1954 in a clay slope at Lodalen near Oslo, originally excavated about 30 years earlier (Fig. 18). Since the slide occurred without any apparent change in external loading, it can be considered to be the result of a gradual reduction in the stability of the slope. Extensive field investigations and laboratory studies were carried out to determine the pore pressure in the slope at the time of failure and the shear parameters of the clay.

Triaxial tests gave the values c' = 250 lb./sq. ft. and $\phi' = 32^0$. An effective stress analysis using equation 14 gave a factor of safety of 1.05, and confirms the validity of the approach to within acceptable limits of accuracy.

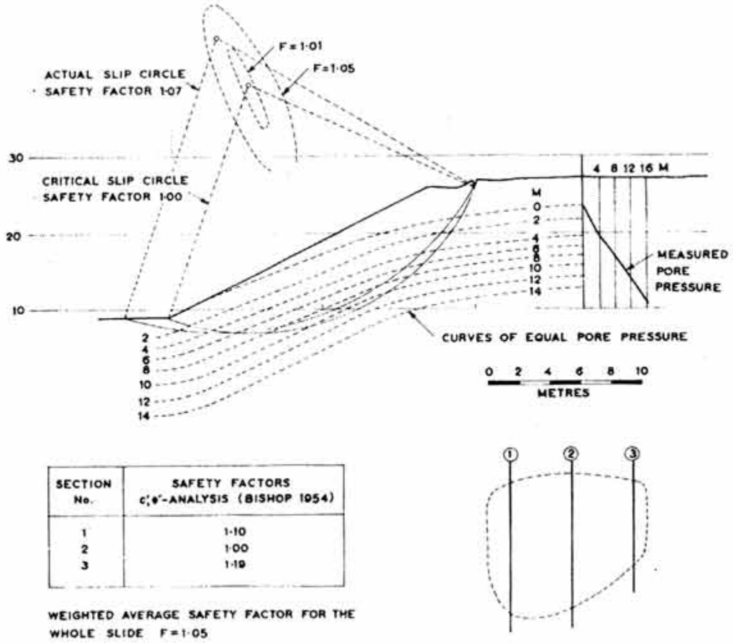
Where the final pore pressures are obtained from a flow net not based on field measurements, allowance should be made for the fact that the permeability of a water laid sediment is generally greater in a horizontal direction (Sevaldson 1956). The highest wet season values obviously represent the most critical conditions.

The excavation of cuts in stiff fissured and weathered clays presents some special problems which have been discussed in detail by Terzaghi, 1936 a; Skempton, 1938 c; Henkel and Skempton, 1955; Henkel, 1957; etc. The reduction in stress enables the fissures to open up and they will then represent weak zones which a sliding surface will tend to follow. The fissures will also increase the bulk permeability of the clay (an increase of about 100 times is reported by Skempton and Henkel, 1960) so that the pore pressure rise leading to the long term equilibrium state will occur more rapidly.

The presence of fissures is reflected in the factors of safety obtained using the effective stress analysis with field values of pore pressure and values of c' and ϕ' measured in the laboratory on 1-1/2" diameter samples. An analysis of three long-term cutting failures in London clay by Henkel (1957), using the laboratory values of c' = 250 lb./sq. ft. and $\phi' = 20^0$, gave factors of safety of 1.32, 1.35, and 1.18. Putting c' = 0 gave values of 0.78, 0.81, and 0.82 respectively and obviously underestimated the factor of safety. If the value of c' required to give a factor of safety of 1 is plotted against time after construction (Henkel, 1957; De Lory, 1957), it is found that the value of c' shows a definite correlation with time (Fig. 19) and, as will be seen in section 6(c), appears to approach zero in natural slopes on a geological time scale.

An effective stress analysis based on a value of c' related empirically with time for each clay type will obviously give a close approximation to the correct factor of safety. For large scale work and for remedial measures on active slips where the large strains tend to reduce c', it is prudent to ensure a factor of safety of at least 1 with c' = 0.

s. This method has also been given since 1956 by Skempton in his lectures at Imperial College.



SECTION No.	SAFETY FACTORS c',φ'-ANALYSIS (BISHOP 1954)
1	1·10
2	1·00
3	1·19

Fig. 18.—Long term failure in a cut at Lodalen (after Sevaldson, 1956).

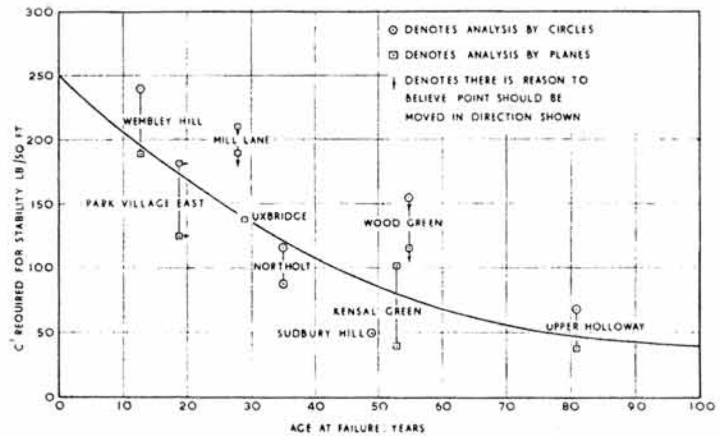


Fig. 19.—Long term failures in stiff-fissured London clay: Correlation of apparent cohesion c' required in effective stress analysis with age of cut at failure (after De Lory, 1957).

The mechanism of the drop in c' before a failure is initiated is not clearly understood, but may be associated with stress concentrations due to the presence of fissures, the progressive spread of an overstressed zone in a soil which tends to dilate and absorb water on shear, and the effect of cyclical fluctuations in effective stress due to seasonal water level changes. On the limited evidence so far available from Lodalen and Selset (see section 6(c)) it does not appear to occur in any marked way in non-fissured clays.

In temporary work, where the end of construction condition is of primary interest, the factor of safety may obviously be calculated by using the $\Phi_u = 0$ analysis and the undrained shear strength. This method may also be used with advantage where it is necessary to check that the initial factor of safety is not lower than the long term value, as it avoids the necessity of explicitly determining the stress distribution and pore pressure values at the end of construction. Four examples of its use are given in Table III.

Also included in Table III is an example of the use of the $\Phi_u = 0$ method for end of construction conditions in a cut in London clay, which led to an overestimate of the factor of safety by 70%. Whether this is simply a consequence of the opening of fissures due to stress release, or due to changes in pore pressure even in the short period of excavation due to the high bulk permeability, is not yet clear. The reduction in strength at low stresses is apparent in stiff fissured clay even in undrained tests in the triaxial apparatus (Fig. 20), but hardly appears adequate to account for the 70% error. A conservative factor of safety must obviously be used in similar cases, and the rapid adjustment to the equilibrium pore pressure condition must be allowed for in prolonged construction operations.

(c) Natural Slopes.—

Natural slopes represent the ultimate long term equilibrium state of a profile formed by geological processes. The pore pressures are controlled by the prevailing ground water conditions which correspond to steady seepage, subject to minor seasonal variations in ground water level. Natural slopes therefore fall into class (a) in which the pore pressure is an independent variable.

In principle the analysis is the same as that of the long term equilibrium of a cut or excavation. However the pore pressures will have already reached their equilibrium pattern which can be ascertained from piezometer measurements in the field; and the natural processes of softening, leaching etc., will have already reached an advanced stage. An analysis based on laboratory tests of this material would therefore be expected to lead to close agreement with observed slopes in limiting equilibrium.

Relatively few natural slopes in limiting equilibrium have yet been analysed in terms of effective stress, but some representative examples are collected in Table IV. The two cases involving intact clays, Drammen and Selset, are being examined in greater detail, but the preliminary values of factor of safety of 1.15 and 1.03 respectively show that the method can be used with reasonable confidence. Had c' tended to zero the Selset slope would have shown a factor of safety of less than 0.7, for example, which is outside the limit of experimental error.

However, in stiff-fissured clays special account has to be taken of the progressive reduction in the value of c', which appears eventually to approach zero in the failure zone, since the shear strains and water content change associated with failure are very localized and tests on the bulk of the soil do not reveal the decrease in c'.

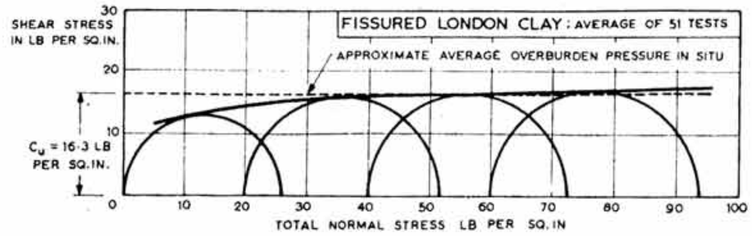


Fig. 20.—The strength of stiff-fissured London clay in undrained tests (after Bishop and Henkel, 1957).

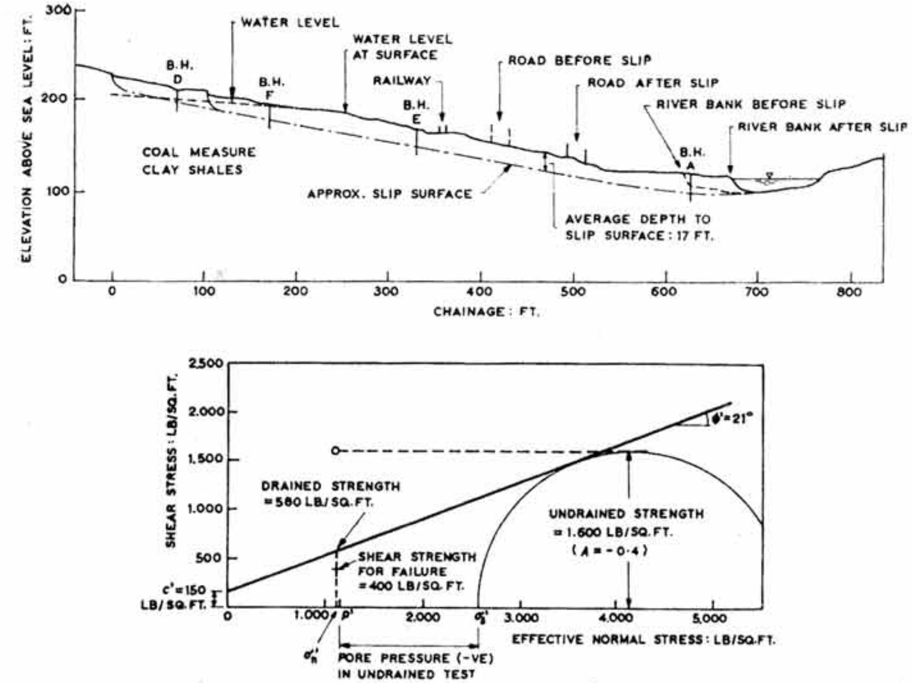


Fig. 21.—Natural slope failure in stiff-fissured clay at Jackfield (after Henkel and Skempton, 1955): Cross-section and shear strength data. Shear strength for limiting equilibrium, 400 lb./sq. ft. (F = 1.0). Undrained shear strength (c' = 150 lb./sq. ft. $\Phi' = 21^o$ A = -0.4), 1.600 lb./sq. ft. (F = 4.0). Drained shear strength (c' = 150 lb./sq. ft. $\Phi' = 21^o$), 580 lb./sq. ft. (F = 1.45). Drained shear strength (c' =0 $\Phi' = 21^o$), 430 lb./sq. ft. (F = 1.07).

Table IV.—Failure of Natural Slopes: Effective Stress Analysis.

Location	Soil type	Data of Clay					Factor of safety: c', Φ' analysis	Reference
		W	LL	PL	PI	$\frac{W-PL}{PI}$		
Drammen	Normally Consolidated (Intact)	31	30	19	11	1.09	1.15	Bjerrum and Kjærnsli 1957
Selset	Overconsolidated (Intact)	13	26	13	13	0	1.03	Imperial College
Jackfield	Overconsolidated (stiff-fissured)	20	45	20	25	0	1.45 (1.07 with $c' = 0$)	Henkel and Skempton 1955

Table V.—Long Term Failures in Cuts and Natural Slopes: $\Phi_u = 0$ Analysis (after Bjerrum and Kjaernsli, 1957).

1. Overconsolidated, fissured clays

Locality	Type of slope	Data of clay					Safety factor, $\Phi_u = 0$ analysis	Reference
		W	LL	PL	PI	$\frac{W-PL}{PI}$		
Toddington	Cutting	14	65	27	38	-0.34	20	Cassel, 1948
Hook Norton	Cutting	22	63	33	30	-0.36	8	Cassel, 1948
Folkestone	Nat. slope	20	65	28	37	-0.22	14	Toms, 1953 b
Hullavington	Cutting	19	57	24	33	-0.18	21	Cassel, 1948
Salem, Virginia	Cutting	24	57	27	30	-0.10	3.2	Larew, 1952
Walthamstow	Cutting	–	–	–	–	–	3.8	Skempton, 1942
Sevenoaks	Cutting	–	–	–	–	–	5	Toms, 1948
Jackfield	Nat. slope	20	45	20	25	0.00	4	Henkel/Skempton, 1955
Park Village	Cutting	30	86	30	56	0.00	4	Skempton, 1948 c
Kensal Green	Cutting	28	81	28	53	0.00	3.8	Skempton, 1948 c
Mill Lane	Cutting	–	–	–	–	–	3.1	Skempton, 1948 c
Bearpaw, Canada	Nat. slope	28	110	20	90	0.09	6.3	Peterson, 1952
English Indiana	Cutting	24	50	20	30	0.13	5.0	Larew, 1952
SH 62, Indiana	Cutting	37	91	25	66	0.19	1.9	Larew, 1952

2. Overconsolidated, intact clays

Tynemouth	Nat. slope	–	–	–	–	–	1.6	Imperial College
Frankton, N.Z.	Cutting	43	62	35	27	0.20	1.0	Murphy, 1951
Lodalen	Cutting	31	36	18	18	0.72	1.01	N.G.I.

3. Normally consolidated clays

Munkedal	Nat. slope	55	60	25	35	0.85	0.85	Cadling/Odenstad, 1950
Säve	Nat. slope	–	–	–	–	–	0.80	Cadling/Odenstad, 1950
Eau Brink cut	Cutting	63	55	29	26	1.02	1.02	Skempton, 1945
Drammen	Nat. slope	31	30	19	11	1.09	0.60	N.G.I.

The landslide at Jacksfield provided a good example of the application of the effective stress analysis to a natural slope in stiff-fissured clay (Henkel and Skempton, 1955) and is illustrated in Fig. 21. The slope of the hillside is 10.5^0, and when the slip took place in the winter 1951-52, a soil mass 600 feet by 700 feet and 17 feet in thickness moved gradually downward about 100 feet.

The calculated average shear stress in the clay was about 400 lb./sq. ft. Drained tests on undisturbed samples gave $c' = 150$ lb./sq. ft. and $\Phi' = 21^0$, which with the observed pore pressures gave a shear strength of 580 lb./sq. ft. and a factor of safety of 1.45. Putting $c' = 0$ gave a shear strength of 430 lb./sq. ft. and a factor of safety of 1.07.

For natural slopes in stiff fissured clays it therefore appears necessary to use $c' = 0$ in the effective stress analysis. This is confirmed by observations made by Skempton and De Lory (1957) on the maximum stable natural slope found in London clay, and by Suklje (1953 a and b), and Nonveiller and Suklje (1955) in other fissured materials. It is interesting to speculate on whether the drop in c' is due to the fissures, or whether both are due to some more fundamental difference in the stress-strain-time relationships between the fissured and intact clays.

A second class of soil which gives rise to special problems includes very sensitive or quick clays. These clays show almost no strength in the remoulded state, and they will therefore tend to flow as a liquid if a slide occurs. A small initial slip in a slope may therefore have catastrophic consequences as the liquified clay will flow away and will not form a support for the exposed clay face, with the result that the whole of an otherwise stable slope may fail in a series of retrogressive slips taking place under undrained conditions.

A factor which affects the quantitative analysis in the case of quick clays is the influence of sample disturbance on the values of c' and Φ' measured in the laboratory. Soft clays of low plasticity are very sensitive to disturbance and reconsolidation in the triaxial test is always accompanied by a reduction in water content. Particularly where the initial water content is above the liquid limit laboratory tests appear to overestimate the value of Φ'. The investigation of a recent slide in quick clay in Norway has given a value of Φ' calculated from the statics of the sliding mass which is less than 50% of the value measured in the triaxial test.

The occurrence of quick clays is limited to certain well defined geological conditions, and where they are encountered special precautions in sampling, testing and analysis are always necessary (Holmsen, 1953; Rosenqvist, 1953; Bjerrum, 1954a and 1955c).

The application of laboratory tests to the stability of natural slopes raises two general matters of principle. The time scales of the load application are so different in the laboratory and in the field that it is perhaps surprising that satisfactory agreement between the results can be obtained at all. Laboratory results quoted by Bishop and Henkel (1957), and Bjerrum, Simons and Torblaa (1958) indicate that under certain conditions Φ' may have a lower value at low rates of loading. This effect may be partly offset by the fact that the worst ground water conditions which touch off the slip are only of seasonal occurrence; and by factors such as the effect of plane strain on the value of Φ' and the omission of 'end effects' in the stability analysis. It is also well known that considerable creep movements occur in slopes still classed at stable. However, with the exceptions noted, the overall correlation between laboratory and field results is quite acceptable from a practical point of view.

Secondly, it may well be asked why the factor of safety cannot be calculated with equal accuracy using the $\Phi_u = 0$ analysis and the undrained strength of samples from the slope where pore pressure and water content equilibrium have been attained. A large number of case records of slides in both natural slopes and cuttings are summarized in Table V and it is evident that as a practical method it is most unreliable, giving values of factor of safety ranging between 0.6 in sensitive clays to 20 in heavily overconsolidated clays.

The fundamental reason for the difference is that in the undrained test the pore pressure is a function of the stress applied during the test, and it is not necessarily equal to the pore pressure in situ. To obtain a factor of safety of 1 for a slope in limiting equilibrium using undrained tests would require that the same pore pressure should be set up in the sample when the in situ normal and shear stresses were replaced. This is in general prevented by the irreversibility of the stress-strain characteristics of the soil and by the changes in the principal stress directions. The latter occur even with in situ tests.

The position is made worse by the fact that the water content changes both in overconsolidated and in sensitive clays are very localized at failure and samples which do not pick up these layers can have little bearing on the stability analysis. A sample from the 2 inch thick slip zone at Jackfield in which large strains had occurred was found to have a water content 10% above the adjacent clay and an undrained strength within 12% of that required for a factor of safety of 1 (Henkel and Skempton, 1955). This layer was difficult to find and sample, and as clay outside the failure zone had an undrained strength nearly four times as great the method has little predictive value.

(d) Base Failure of Strutted Excavations in Clay.—

During excavation in soft clay, base failure sometimes occurs accompanied by settlement of the adjacent ground. Failures of this type[t] have occurred in excavations for basements, in trenches for water and sewage pipes, and in the shafts for deep foundations.

The construction of temporary excavations is generally carried out sufficiently rapidly for pore pressure changes to be ignored. The change in stress thus occurs under undrained conditions and the stability can be calculated using the $\Phi_u = 0$ analysis and undrained tests.

The factor of safety F against base failure can be derived from the familiar bearing capacity theory, considering the excavation as a negative load. This leads to the expression (Bjerrum and Eide, 1956):

$$F = N_c \cdot c_u/(\gamma D + q) \tag{24}$$

where D denotes depth of excavation

- γ » density of the clay
- c_u » the undrained strength of the clay beneath the bottom of the excavation
- q » the surface surcharge (if any)
- N_c » dimensionless bearing capacity factor depending on shape and depth of excavation.

t. Bottom heave failures can also occur in clay if a pervious layer containing water under sufficient head lies close beneath the excavation. For example see Garde-Hansen and Thernöe 1960, and Coates and Slade 1958.

The analysis of the failure of seven excavations is given in Table VI. The results indicate that in practice an accuracy of within ± 20% can be expected.

It should be noted that this type of failure is not caused by inadequate strutting, but the loads and distortion after its occurrence may initiate a more general collapse.

Table VI.—Base Failure of Strutted Excavations in Saturated Clay: $\Phi_u = 0$ Analysis (after Bjerrum and Eide, 1956).

Site	Dimensions $B \times L$: m	Depth D : m	Surcharge p : tons/sq. m	Density γ : tons/cu. m	Shear strength S : tons/sq. m	Sensitivity	B/L	D/B	N_c theoretical	Safety factor F	Average safety factor
1. Pumping station, Fornebu, Oslo	5.0×5.0	3.0	0.0	1.75	0.75	50	1.0	0.60	7.2	1.03	
2. Storehouse, Drammen	4.8×∞	2.4	1.5	1.90	1.2	5–10	0.0	0.50	5.9	1.16	
3. Pier shaft, Göteborg	ø 0.9	25.0	0.0	1.54	3.5	20–50	1.0	28.0	9.0	0.82	
4. Sewage tank, Drammen	5.5×8.0	3.5	1.0	1.80	1.0	20	0.69	0.64	6.7	0.93	0.96
5. Test shaft (N) Ensjøveien, Oslo	ø 1.5	7.0	0.0	1.85	1.2	140	1.0	4.7	9	0.84	
6. Excavation, Grev Vedels pl., Oslo	5.8×8.1	4.5	1.0	1.80	1.4	5–10	0.72	0.78	7.0	1.08	
7. "Kronibus shaft", Tyholt, Trondheim	2.7×4.4	19.7	0.0	1.80	3.5	40	0.61	7.3	8.5	0.84	

(e) Earth Pressures on Earth Retaining Structures.—

If the displacement of an earth retaining structure is sufficient for the full development of a plastic zone in the soil adjacent to it, the earth pressure will be a function of the shear strength of the soil. This condition is apparently satisfied in much temporary work and in many permanent structures. The distribution of pressure is a function of the deformation of the structure and the soil, and can only be predicted after detailed consideration of the movements involved.

For temporary excavations in intact saturated clay it is generally sufficient to calculate the total earth pressure using the $\Phi_u = 0$ analysis and the undrained shear strength. Justification for this procedure is to be found in the field measurements published by Peck (1942), Skempton and Ward (1952), and Kjaernsli (1958). More recent measurements in soft clay carried out by the Norwegian Geotechnical Institute however indicate that the total earth pressure may exceed the value determined by the $\Phi_u = 0$ analysis and that the ratio of the actual to the calculated load increases with the number of struts used to carry it.

The long term earth pressure is logically computed using the effective stress analysis with values of c' and Φ' taken from drained tests or consolidated-undrained triaxial tests with pore pressure measurements together with the least favourable position of the water table. This will in most cases represent a rise in earth pressure. Few examples are available to confirm this analysis other than of gravity retaining walls which are themselves founded in the same clay stratum. The problem is then in effect one of overall stability since the slip surface passes beneath the wall which is then of little more consequence than one 'slice' in the slices method of analysis (Fig. 22).

A number of failures of this type in stiff fissured London clay have been analysed by Henkel (1957), and here consistent active and passive pressures on the walls have been obtained using the effective stress analysis and observed water levels, together with the reduced values of c' shown in Fig. 19. It should be noted that where the excavation in front of the wall is deep, the presence of the passive pressure is insufficient to prevent the occurrence of progressive softening.

The behaviour of gravity retaining walls can of course throw little light on the end of construction earth pressures in fissured clays. Measurements of strut load have however been made by the Norwegian Geotechnical Institute in a trench in the weathered stiff fissured crust overlying a soft clay stratum (Di Biagio and Bjerrum, 1957; Bjerrum and Kirkedam, 1958). Here the softening appeared to proceed more rapidly than in cuts in London clay, for after only a few months the strut loads corresponded to the value given by the effective stress analysis with $c' = 0$.

The evidence from this cut and the indirect evidence from the Bradwell slip (section 6(b)) indicates that the $\Phi_u = 0$ analysis does not correctly represent the behaviour of stiff fissured clays under decreasing stresses even shortly after excavation. The rapid dissipation of negative pore pressures due to the presence of open fissures is obviously an important factor in temperate climates. Under long term conditions the $\Phi_u = 0$ method is also inapplicable for the reasons given in sections 6(b) and (c).

(f) The Stability of Earth Dams.—

It is impossible to deal adequately with all the stability problems arising in earth dam construction in one short section. However, the most important

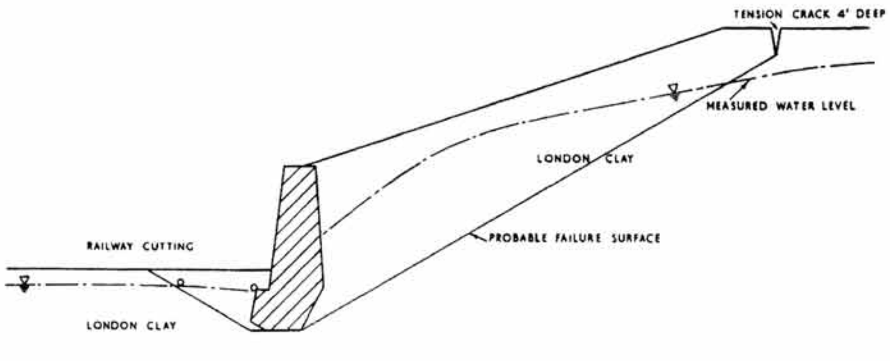


Fig. 22.—Retaining wall failure in stiff-fissured London clay (after Henkel, 1957).

principles may be illustrated by considering the stability of a water retaining dam built mainly of rolled earth fill (Fig. 23).

The stability of the slopes and foundation of an earth dam against shear failure will generally have to be considered under three conditions:

(1) During and shortly after construction,
(2) With the reservoir full (steady seepage), and
(3) On rapid drawdown of the impounded water.

Additional considerations arising from the possibility of failure in a clay foundation stratum are outlined in section 6(h). In this section attention will be limited to the fill.

The stability may be calculated for all three conditions in terms of effective stresses. This involves the measurement of c' and Φ' in the laboratory and an estimate of the pore pressure values at each stage. The use of explicitly determined pore pressures in the analysis enables the field measurements of pore pressure which are made on all important structures to be used as a direct check on stability during and after construction. It also enables the design estimates to be checked against the wealth of pore pressure data now becoming available from representative dams—for example the extensive work of the U.S.B.R. recently summarized by Gould (1959) and special cases such as the Usk dam (Sheppard and Aylen, 1957) and Selset dam (Bishop, Kennard and Penman, 1960).

To work directly with undrained test results expressed in terms of total stress may be unsafe in low dams as it implies dependence on negative pore pressures which will subsequently dissipate; and uneconomical in high dams in wet climates as no account is taken of the dissipation of excess pore pressure during the long construction period.

For the effective stress analysis the values of c' and Φ' are generally obtained from undrained triaxial tests with pore pressure measurement. In earth fill compacted at water contents well above the optimum a series of consolidated-undrained tests with pore pressure measurements may have to be used instead, in order to obtain a sufficient range of effective stresses to define a satisfactory failure envelope.

For the analysis of the condition of long term stability under steady seepage and for the case of rapid drawdown it is necessary to consider the effect of saturation on the values of c' and Φ'. As mentioned in section 4(h), test results show that, in general, the value of Φ' remains almost unchanged. Where c' has an appreciable value in the undrained tests this will decrease. However, tests using the improved techniques described by Bishop (1960) and Bishop, Alpan, Blight, and Donald (1960) have failed to reproduce the high cohesion intercepts previously reported in undrained tests. Provided c' has been accurately measured in either type of test, the differences may only be of significance in important works where the margin of safety is small. Whether c' is likely to become zero in rolled fill on a really long term basis is discussed by Bishop (1958 b) and Terzaghi (1958). Evidence so far does not appear to point to such a reduction.

The principal factors controlling the pore pressure set up during construction are:

(i) The placement moisture content and amount of compaction, and hence the pore pressure parameters;
(ii) The state of stress in the zone of the fill considered, and
(iii) The rate of dissipation of pore pressure during construction.

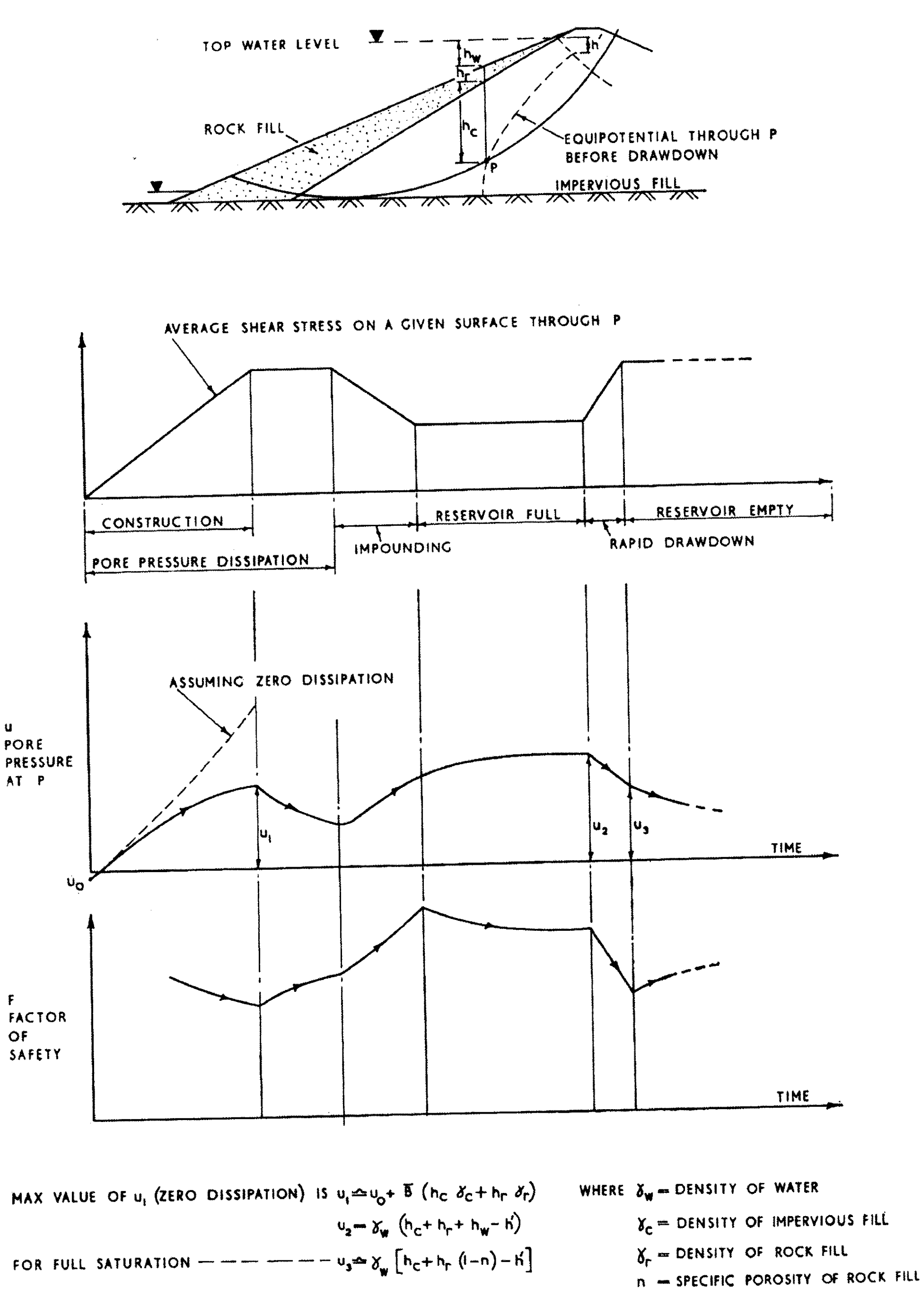


Fig. 23.—The changes in shear stress, pore pressure and factor of safety for the upstream slope of an earth dam.

In section 3 it was shown that the pore water pressure set up under undrained conditions can be expressed in the form:

$$u = u_0 + \bar{B}\,.\,\Delta\sigma_1 \tag{7}$$

In Fig. 24 the values of u_0 and $\bar{B}$ are plotted against water content for a series of samples prepared with the compactive effort used in the standard compaction test. This clearly shows the sensitivity of the value of the initial pore pressure to the placement water content; the importance of this effect, both in design and construction, cannot be overemphasised.

It is usually assumed that the value of σ_1 is equal to the vertical head of soil γ. h above the point considered, although the direction in which σ_1 acts is not necessarily vertical. This is a reasonably satisfactory assumption when averaged around a complete slip surface, but tends to overestimate the pore pressure in the centre of the dam and underestimate it near the toes (Fig. 25, after Bishop, 1952). It enables the pore pressure ratio required for the stability analysis to be expressed, under undrained conditions as

$$r_u = \bar{B} + u_0/\gamma h \tag{9}$$

or, at the higher water contents where $\bar{B}$ is large and u_0 small, more simply as

$$r_u = \bar{B} \tag{10}$$

However, in most earth fills a considerable reduction in the average pore pressure results from dissipation even during the construction period. A numerical method of solving the practical consolidation problem with a moving boundary has been given by Gibson (1958). It should be noted that in many almost saturated soils even a small amount of drainage has a marked effect on the final pore pressure, since it not only reduces the pore pressure already set up, but also reduces the value of $\bar{B}$ under the next increment of load. The theoretical basis of this reduction is discussed by Bishop (1957), and it is confirmed by field results from the Usk dam (Fig. 26a).

As an example of the distribution of pore pressure at the end of construction the contours from the Usk dam are illustrated in Fig. 26b. The effectiveness of the drainage layers placed to reduce the average pore pressure in the fill will be apparent.

The average pore pressure ratio along a potential slip surface at the end of construction may be kept within safe limits either by restricting the size of the impervious zone, by strict placement water content control or by special drainage measures (as at the Usk and Selset dams). Which is the more economical procedure will depend on the climatic conditions and the fill materials available.

For reservoir full conditions the pore pressure distribution may be predicted from the flow net corresponding to steady seepage. Accuracy is difficult to obtain owing to the non-uniformity of the rolled fill and differences in the ratio of horizontal to vertical permeability, so conservative assumptions should be made. However, with properly placed drainage zones the average value of r_u for the downstream slope is generally less than during construction, except in low dams or with rather dry placement.

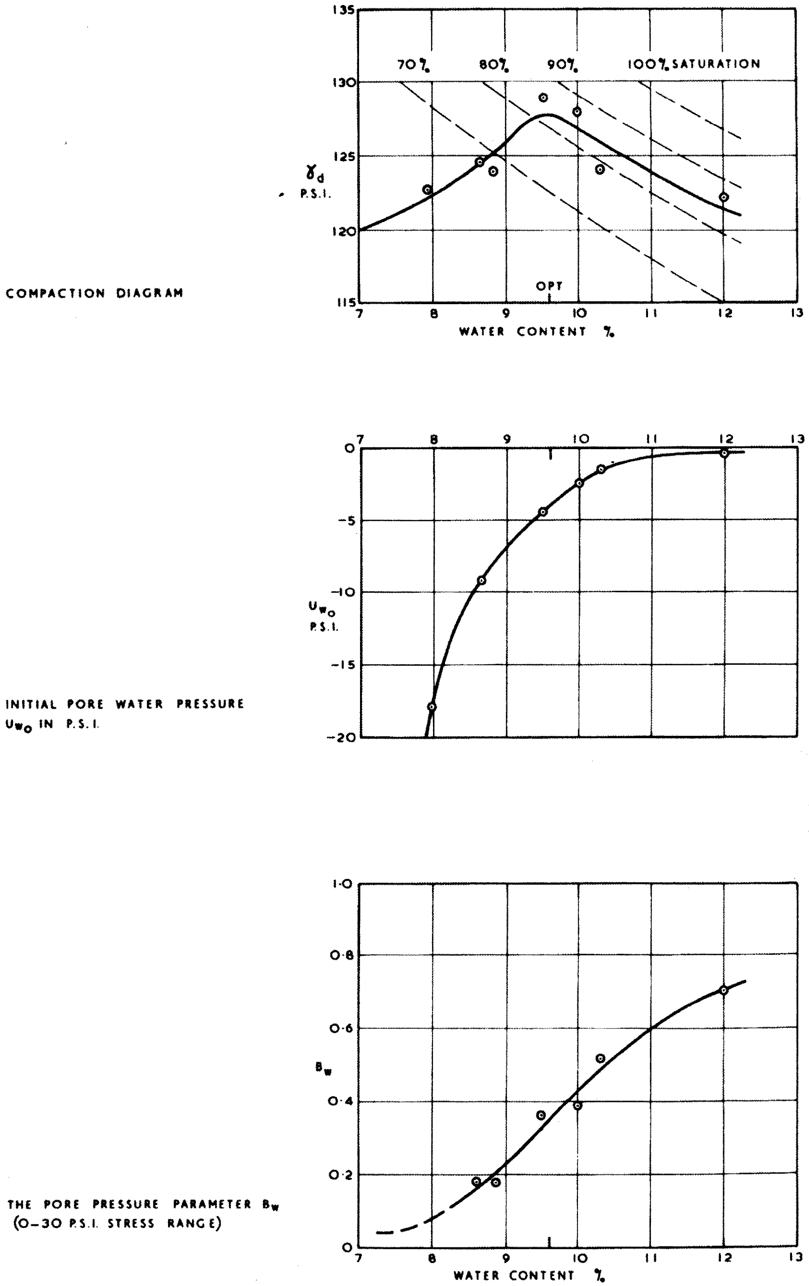


Fig. 24.--Represents Values from Standard Compaction Test under Equal All Round Pressure Increase: Compacted Boulder Clay, Clay Fraction 6%

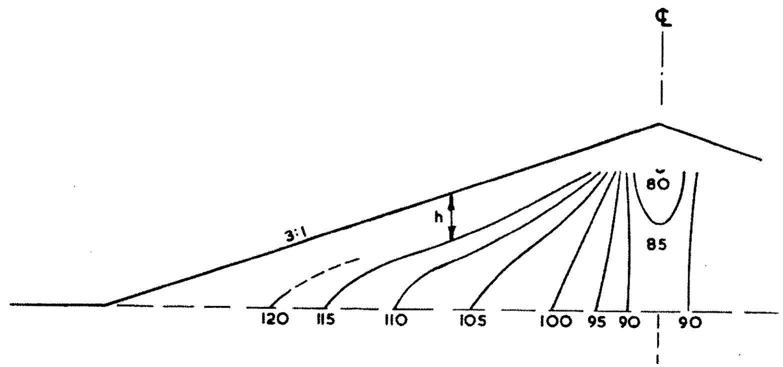


Fig. 25.—Major Principal Stress σ, as a Percentage of γ . h, the Vertical Head of Soil Above the Point Considered (After Bishop, 1952).

A method of predicting the excess pore pressures resulting from rapid drawdown has been proposed by Bishop (1952 and 1954a). In this method the change in pore pressure on drawdown is assumed to take place under undrained conditions and is deduced from the stress change and the pore pressure parameters (see Fig. 23). For saturated fills the value of $\bar{B}$ is taken as 1; the change in the value of σ_1 is due to the removal of the water load from the face of the dam and the drainage of water from the voids of the rockfill.

This method shows reasonable agreement with the results of the field measurements on the Alcova dam (Glover, Gibbs and Daehn, 1948). Two recent cases of very rapid drawdown soon after completion are not in such good agreement, but both involve complicating factors (Bazett, 1958; Paton and Semple, 1960).

Fig. 23 shows diagrammatically the variation in pore pressure and factor of safety for the various phases in the life of the upstream slope of the embankment calculated as described above. The lowest values of factor of safety are usually reached at the end of construction and on rapid drawdown.

For the downstream slope, end of construction and steady seepage are the two critical stages. However, during steady seepage the danger is generally not so much from the pore pressures, which are easily controlled by drainage measures, but from the possibility of piping and internal erosion in the foundation strata, and from crack formation in the fill.

For small earth dams built largely of saturated soft clay, the stability during the construction period can be calculated by the $\Phi_u = 0$ analysis using the undrained strength (Cooling and Golder, 1942). However, where the fill is much stronger than the clay foundation strata satisfactory results are not obtained (for example, Golder and Palmer, 1955) for the reasons given in section 6(a). The long term stability can of course be determined only by the effective stress analysis.

(g) Stability of Slopes in Sand and Gravel on Drawdown.—

In relatively pervious soils of low compressibility the distribution of pore pressure on drawdown is controlled by the rate of drainage of pore water

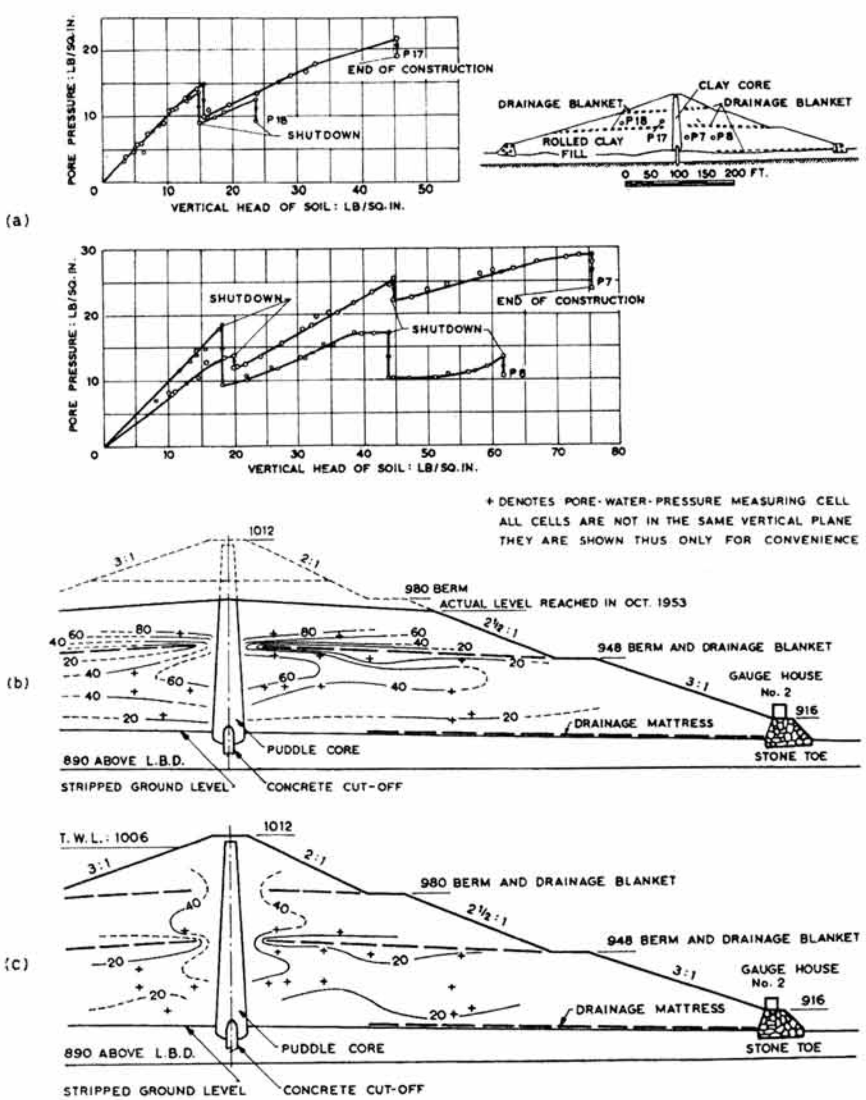


Fig. 26.—Field measurements of pore pressure from the Usk dam: (a) The reduction in pore pressure and in the value of $\bar{B}$ due to pore pressure dissipation (after Bishop, 1957); (b) and (c) contours of pore pressure expressed as a percentage of the vertical head of soil in October, 1953 and October 1954 (after Sheppard and Aylen, 1957).

from the soil. This condition can be represented by a series of flownets with a moving boundary as shown by Terzaghi (1943) and Reinius (1948).

The flow pattern is a function of the ratio of drawdown rate to permeability and the values of the pore pressures to be used in the stability analysis can be taken from the appropriate flow-net. The influence of the greater permeability in the horizontal direction is considerable, but, in one case examined, tended to increase rather than reduce the factor of safety.

The values of c' and Φ' are obtained from drained tests, c' approaching zero for free-draining materials.

An example of a drawdown failure in Thames gravel is shown in Fig. 27. The initial slope of the gravel was 33° and the permeability about 0.05 cm/sec. The value of Φ' in the loose state was 36°. Failure occurred when the pool was lowered at a rate of about 1 foot per day (Bishop, 1952).

(h) The Stability of a Clay Foundation of an Embankment where the Rate of Construction Permits Partial Consolidation.—

It is not uncommon in earth dam construction to encounter geological conditions in which the foundation strata include a soft clay layer at or near the surface, of sufficient extent to be likely to lead to failure in an embankment having conventional side slopes (for example Cooling and Golder, 1942; McLellan, 1945; Bishop, 1948; Skempton and Bishop, 1955; Bishop, Kennard and Penman, 1960). It is then necessary to assess the economics and practicability of a number of alternative solutions. The soft layer may be excavated, if its depth and ground water conditions permit; or an embankment with very flat slopes may be accepted, its factor of safety being calculating using the $\Phi_u = 0$ method which assumes zero drainage. Alternatively, account may be taken of the dissipation of pore pressure which occurs due to natural drainage (for example, Bishop, 1948) or due to special measures, such as vertical sand drains, designed to accelerate consolidation (for example, Skempton and Bishop, 1955; Bishop, Kennard and Penman, 1960). In this case an effective stress analysis is used.

An expression for the initial excess pore pressure in a saturated soft clay layer where B = 1 has been obtained by Bishop (1952):

$$\Delta u = \Delta p + p_o \,.\, [(1-K)/2] + (2A-1)\sqrt{p_o^2[(1-K)/2]^2 + \tau^2} \qquad (25)$$

where Δp denotes change in total vertical stress due to the fill,
τ » shear stress along the layer set up by the fill,
p_o » initial vertical effective stress,
and Kp_o » initial horizontal effective stress in clay layer.

This expression illustrates the dependence of pore pressure on the change in shear stress as well as on the change in vertical stress, though the latter predominates in most practical cases. To avoid the error in the estimate of A which may arise from the change in void ratio on reconsolidation of undisturbed samples, the value of A may be deduced from the relation between the undrained strength and the effective stress envelope, using an assumed value of K. The relationship is given by Bishop (1952).

The estimate of the rate of dissipation of pore pressure is based on the theory of consolidation. It is here that the greatest uncertainity arises, especially in stratified deposits, and field observations of pore pressure are advisable on important works.

Fig. 27.—Drawdown failure in Thames Valley gravel (after Bishop, 1952).

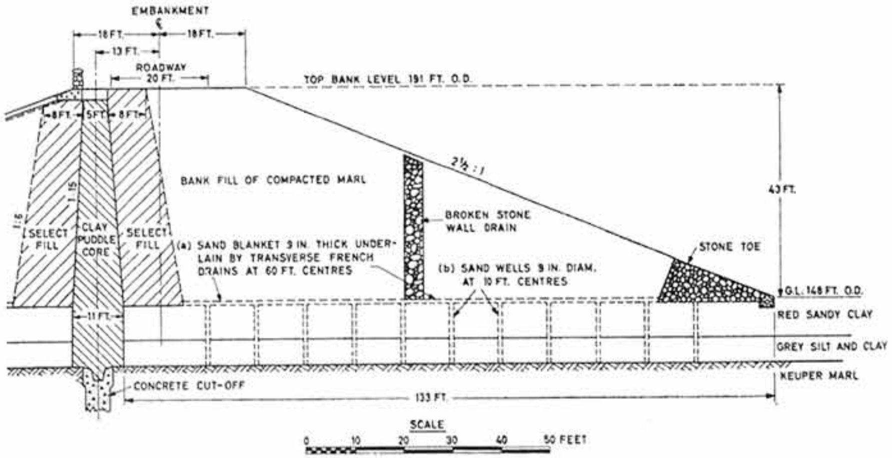


Fig. 28.—Downstream slope of the Chew Stoke dam showing vertical sand drains to accelerate dissipation of pore pressure in soft clay foundation (after Skempton and Bishop, 1955).

The values of c' and ϕ' are taken from drained tests or consolidated undrained tests with pore pressure measurement.

The downstream slope of the Chew Stoke dam (Fig. 28) which had a factor of safety against a foundation failure of 0.8 using the $\phi_u = 0$ analysis was safely constructed using a sand drain spacing designed to give a factor of safety of 1.5 (Skempton and Bishop, 1955). Field observations of pore pressure indicated that the actual factor of safety was rather higher than 1.5 owing to the greater horizontal permeability resulting from stratification of the clay. The Selset dam, founded on a boulder clay with little apparent stratification, showed a smaller difference between predicted and observed pore pressure values (Bishop, Kennard and Penman, 1960).

(i) Some Special Cases.—

In the examples described above the variation in safety factor with time was either a steady increase or a steady decrease during the period from the end of construction until the pore pressures reached an equilibrium condition. The following examples will illustrate that under certain conditions we may temporarily encounter a lower factor of safety at an intermediate stage.

Such cases are obviously very dangerous, as a failure might well occur some weeks or months after the completion of construction, in spite of the fact that it had been ascertained that the factor of safety was adequate in both the initial and final stages. The basic reason in each case is that the redistribution of excess pore pressure which occurs during the consolidation process may lead to a temporary rise in pore pressure outside the zone where the load is applied.

An interesting example is the stability of a river bank in a clay stratum under the action of the excess pore pressure set up by pile driving for a bridge abutment in the vicinity (Bjerrum and Johannessen, 1960). The changes in pore pressure with time are shown diagrammatically in Fig. 29a for two points, one in the centre of the pile group and one outside it, but beneath the slope. The excess pore pressures set up by pile driving will dissipate laterally as well as vertically, particularly in a water-laid sediment where the horizontal permeability tends to be greater than the vertical.

The exact magnitude of the effect is difficult to predict theoretically, and in this case field measurements of pore pressure were used to regulate the progress of pile driving. The most critical distribution of pore pressure occurred shortly after piling was completed (Fig. 29b). The factor of safety of the slope was calculated using the effective stress analysis with c' = 200 lb./sq. ft. and $\phi' = 27^{\circ}$, and dropped from an initial value of 1.4 to 1.15 after piling, assuming low water in the river in each case.

A case in which the spread of pore pressure led to an embankment failure some time after construction has been analysed in detail by Ward, Penman and Gibson (1955). The clay foundation on which the embankment was built included two horizontal layers of peat. Due to the relatively higher permeability of the peat, the redistribution of pore pressure after the end of construction resulted in temporary high pore pressures in the peat on both sides of the embankment, where initially the pore pressures were low. Because of the difference between the void ratio pressure curves for consolidation and swelling, the reduction in effective stress due to the presence of an additional volume of pore water is much greater than the increase in effective stress in the zone from which this volume has migrated. An overall decrease in effective stress along a critical slip path can thus occur (Fig. 30) at an

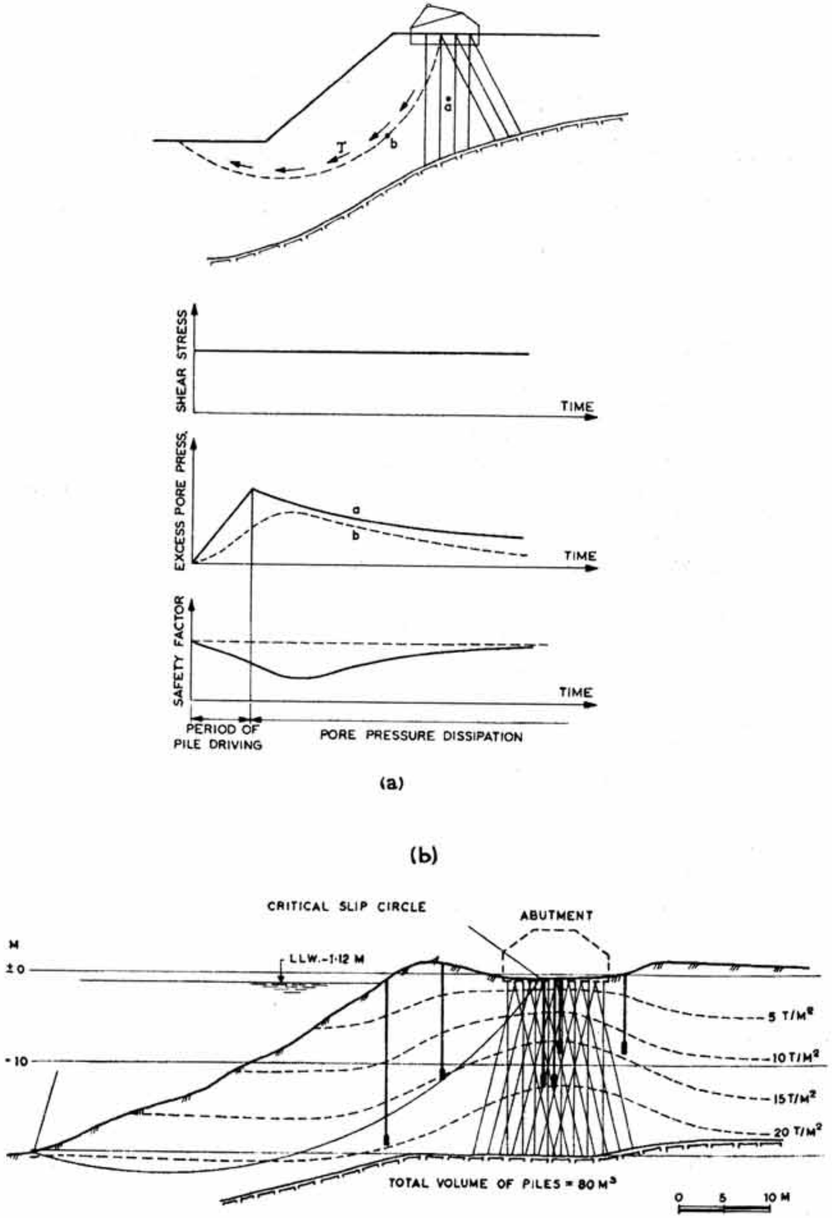
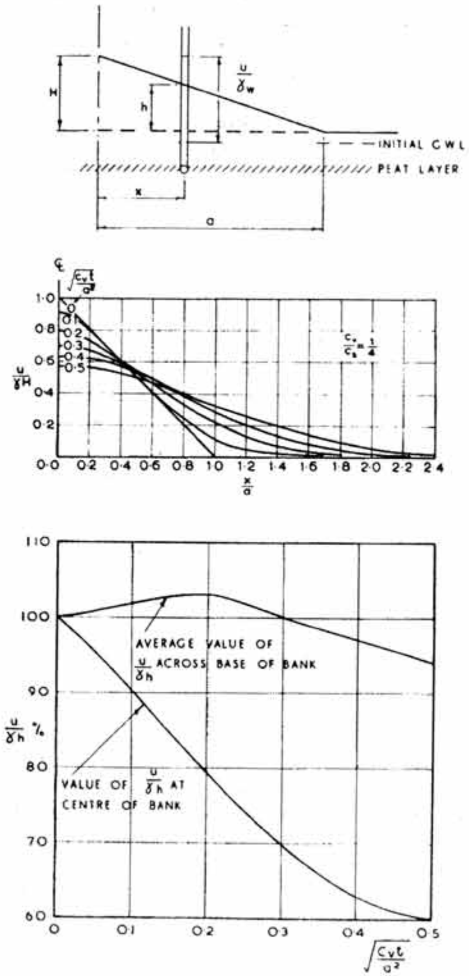


Fig. 29.—The effect of pore pressure set up by pile driving on the stability of an adjacent clay slope: (a) Changes in pore pressure and factor of safety with time: diagrammatic, (b) observed pore pressures just after the completion of pile driving (after Bjerrum and Johannessen, 1960).

intermediate stage, although in the long term equilibrium state the bank foundation would have been stable.[u]

Other engineering operations may result in a similar danger, such as the rapid construction of an embankment or stockpile even some way back from a river bank, cut or quay wall close to limiting equilibrium, and the driving of ordinary piles or screw piles through the clay slopes of rivers or harbours. In such cases an awareness of the danger will either lead to a modification of the operation, or to the use of field measurements of pore pressure as a control while it is carried out.



(a) Embankment on a clay foundation with a horizontal peat layer—diagrammatic;

(b) changes in pore pressure with time, as a fraction of $\gamma . H$ where γ is density of fill;

(c) effect of redistribution on maximum and average pore pressure for $c_v / c_s = 1/4$ where c_v is coefficient of consolidation and c_s is coefficient of swelling (after Ward, Penman, and Gibson, 1954).

Fig. 30.

u. Other failures of this type are described by Terzaghi and Peck (1948).

It is probable that a number of failures attributed to 'creep' could more correctly be attributed to the redistribution of pore pressure which occurs after construction.

(j) Design in Earthquake Areas.—

The analysis of the stability of a structure or dam in an area subject to earthquakes raises special problems which are outside the scope of this paper.

It is, however, known that a transient load will leave residual excess pore pressures which may be positive or negative depending on the void ratio and stress history of the soil (Bishop and Henkel, 1953). A possible way of evaluating the stability under earthquake conditions may therefore be to use a series of consolidated-undrained tests in which the stress ratio during consolidation is chosen to represent the conditions prevailing in the field before the earthquake. The sample is then subjected to a series of small variations in deviator stress under undrained conditions corresponding to the additional seismic stresses. The magnitude of the residual pore pressure and the additional strain will indicate the likelihood of failure under field conditions.

A discussion of the additional shear stresses likely to be set up in earthquake areas is given by Ambraseys (1959).

7. CONCLUSIONS

The discussion and case records presented in this paper point to four main conclusions:

(I) The effective stress analysis is a generally valid method for analysing any stability problem and is particularly valuable in revealing trends in stability which would not be apparent from total stress methods.

Its application in practice is limited to cases where the pore pressures are known or can be estimated with reasonable accuracy. These include all the class (a) problems, such as long term stability and drawdown in incompressible soils, where the pore pressure is controlled by ground water conditions or by a flow pattern. It is also applicable to both class (a) and class (b) problems where field measurements of pore pressure are available.

Those class (b) problems where the magnitude of the pore pressure has to be estimated from the stress distribution and the measured values of the pore pressure parameters can often be solved more simply by the $\phi_u = 0$ analysis. However, this alternative gives no indication of the long term stability and does not enable account to be taken of dissipation of pore pressure during construction, which may contribute greatly to economy in design.

(II) Where a saturated clay is loaded or unloaded at such a rate that there is no significant dissipation of the excess pore pressures set up, the stability can be determined by the $\phi_u = 0$ analysis, using the undrained strength obtained in the laboratory or from in-situ vane tests.

This method is very simple and reliable if its use is restricted to the conditions specified above. It is essentially an end of construction method, and in the majority of foundation problems, where the factor of safety increases with time, it provides a sufficient check on stability. For cuts, on the other hand, where the factor of safety generally decreases with time, the $\phi_u = 0$ method can be used only for temporary work and the long term stability must be calculated by the effective stress analysis.

(III) The two methods of analysis require the measurement of the shear strength parameters c' and ϕ' in terms of effective stress on the one hand and the undrained shear strength c_u under the stress conditions obtaining in the field on the other.

For saturated soils the values of c' and ϕ' are obtained from drained tests or consolidated undrained tests with pore pressure measurement, carried out on undisturbed samples. The range of stresses at failure should be chosen to correspond with those in the field. Values measured in the laboratory appear to be in satisfactory agreement with field records with two exceptions. In stiff fissured clays the field value of c' is lower than the value given by standard laboratory tests; in some very sensitive clays the field value of ϕ' is lower than the laboratory value.

For partly saturated soils the values of c' and ϕ' are obtained from undrained or consolidated-undrained tests with pore pressure measurement, or from drained tests. Provided comparable testing procedures are used the differences between the values of ϕ' obtained appear not to be significant from a practical point of view. The values of c' will be slightly influenced by moisture content differences resulting from the different procedures.

The undrained shear strength c_u is obtained from undrained triaxial tests on undisturbed samples (or from unconfined compression tests, except on fissured clays) and from vane tests in situ. It cannot be obtained, without risk of error on the unsafe side, from consolidated-undrained tests where the sample is reconsolidated under the overburden pressure. The error is serious in normally consolidated clays of low plasticity, and though it can be minimised by consolidating under the stress ratio obtaining in the field, the effect of reconsolidation on the void ratio cannot be avoided.

For this same reason it is probably more realistic to calculate the value of the pore pressure parameter A for undisturbed soil from the relationship between the undrained strength of undisturbed samples and the values of c' and ϕ', rather than to measure it in a consolidated-undrained test.

(IV) The reliability of any method can ultimately be checked only by making the relevant field measurements when failures occur or when construction operations are likely to bring a soil mass near to limiting equilibrium. The number of published case records in which the data is sufficiently complete for a critical comparison of methods is still regrettably small.

ACKNOWLEDGMENTS

The work of K. Terzaghi, Arthur Casagrande, A. W. Skempton and the late D. W. Taylor has contributed so much to the background of any study of shear strength and stability that specific references in the text are inadequate acknowledgment. The authors would also like to express their gratitude to their colleagues at Imperial College and the Norwegian Geotechnical Institute for valuable comments on the manuscript.

APPENDIX I.—THE USE OF THE PARAMETER ϕ_{cu}

In section 4b reference has been made to errors likely to arise in applying in the field the relationship between undrained strength and consolidation pressure obtained in the laboratory from the consolidated-undrained test.

Two inherent errors have been referred to: The effect of reconsolidation after sampling on the void ratio and on the value of the pore pressure parameter A; and the error arising from consolidation under a stress ratio different from that obtaining in the ground. A further error may arise from the way in which the results are introduced into the stability analysis.

This point is illustrated in Fig. 31 (after Bishop and Henkel, 1957). The test is usually performed by consolidating the sample under a cell pressure p, and then causing failure under undrained conditions by increasing the axial stress. The total minor principal stress at failure (σ_3) is thus equal to p; the total major principal stress is $(\sigma_1)_{cu}$. The slope of the envelope to a series of total stress circles obtained in this manner (Fig. 31a) is denoted ϕ_{cu}, the angle of shearing resistance in consolidated undrained tests, and is about one half of the slope of the effective stress envelope (denoted by ϕ') for normally consolidated samples. This relationship between shear strength and total normal stress can only be used in practice if the identity between consolidation pressure and total minor principal stress imposed in the test also

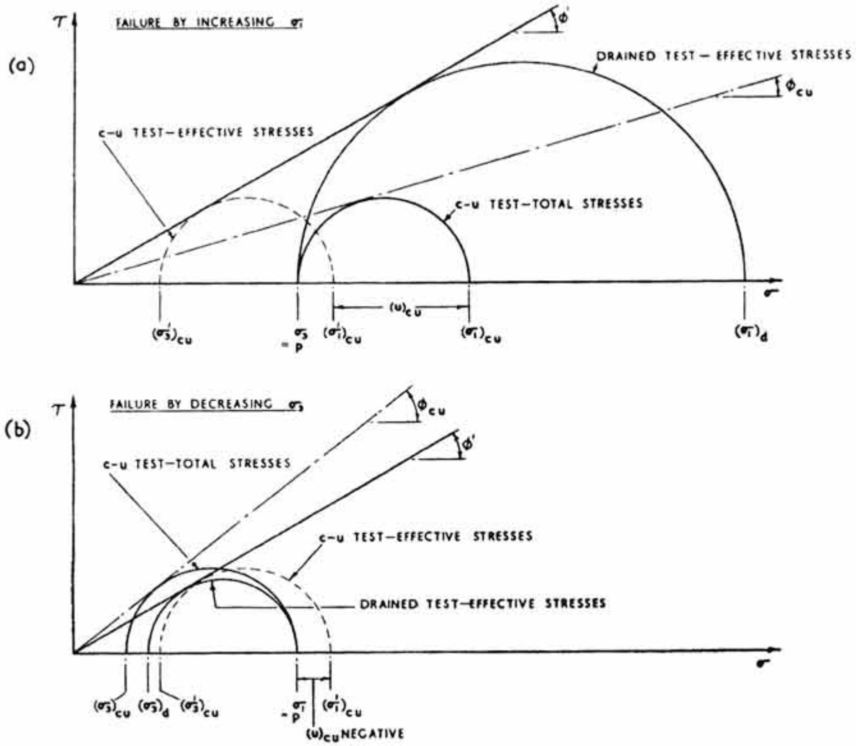


Fig. 31.—The consolidated-undrained test on a saturated cohesive soil in terms of total and effective stresses: (a) Failure by increasing major principal stress σ_1; (b) failure by decreasing minor principal stress σ_3 (after Bishop and Henkel, 1957).

applies around the slip surface considered. Passive earth pressure appears to be the only case in which this is approximately true.

Had the failure been caused by holding σ_1 constant and equal to p and decreasing the total minor principal stress σ_3, the undrained strength would have remained the same, and a radically different value of ϕ_{cu} would have been obtained, Fig. 31b. The relationship between shear strength and total normal stress would then approximate to the case of active earth pressure.

The general use of ϕ_{cu} (defined in Fig. 31a) as an angle of shearing resistance in conventional stability analyses is therefore likely to lead to very erroneous results, even if the samples are anisotropically consolidated. If σ_3 increases during the undrained loading (as in foundation problems) the factor of safety will be overestimated; if σ_3 decreases, as in the excavation of a cutting, the error may lead to an under-estimate of the factor of safety.

The most logical solution appears to be to plot contours of undrained strength in terms of the consolidation pressure in the ground prior to the undrained loading to be examined, and then to use the $\phi_u = 0$ analysis. This method is of course limited to the end of construction analysis, in which it is assumed that insufficient time has elapsed for consolidation or swelling to occur.

In rapid drawdown analyses suggested by Terzaghi (1943) and Lowe and Karafiath (1959) the undrained strength is related to the effective normal stress on the potential failure plane before drawdown. However, unless the samples are failed by reducing the stresses, there is a danger of overestimating the undrained strength of compacted samples which are difficult to saturate fully in the laboratory.

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